Generalised Connectedness Measures of the Global Economy*

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Abstract

The GVAR framework introduced by PSW largely circumvents the traditional curse of dimensionality associated with the limits imposed by the range and frequency of existing macroeconomic datasets. However, ironically, it introduces in turn a secondary curse of dimensionality whereby the limits of an individual's ability to process and interpret reported statistics become the binding processing constraint. Extending the univariate or the single market approach advanced by Diebold and Yilmaz (2009, 2011) we propose a family of Generalised Connectedness Measures (GCMs for the multi-country and the multi-variable global VAR model. By computing GCMs at the level of individual variables (V-GCMs) or variable groups (G-GCMs), at the country-level (C-GCMs) and at the regional level (R-GCMs), we are able to map both the relative strength and direction of the connections in the global economy in a parsimonious and readily interpretable manner. In this way, we are able to unlock more of the potential of existing techniques for the estimation of large-scale economic models. Moreover, GCMs provide a rich source of information that may be used to supplement the range of preliminary descriptive statistics that are currently reported in much applied empirical research. Our methodology therefore represents a valuable addition to the array of techniques that may be employed in exploratory data analysis.

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1 Introduction

Applied macroeconomists, practitioners and policymakers alike have long sought to model the complex interconnections between entities in economic systems. These entities may take the form of firms, markets, sovereign states or economic blocs, for example, while the system under scrutiny may range from a relatively tightly focused analysis of leading stock markets to a broad macro-financial model of the global economy, for example. Many such analyses have employed combinations of impulse response analysis and forecast error variance decomposition. In particular, recently, Diebold and Yilmaz (2009) provide a simple and intuitive measure of interdependence or spillovers of asset returns or volatilities by employing vector autoregressive (VAR) models and aggregating orthogonalised (forecast error) variance decompositions across markets. However, results based on Cholesky-decomposition are well-established to be sensitive to ordering, as Cholesky-factor exact identification amounts to the assumption of a particular recursive ordering, and cannot be tested empirically. In this regard, Diebold and Yilmaz (2011) propose the order-invariant generalised variance decompositions, originally advanced by Pesaran and Shin (1998), to develop the measures of connectedness among different financial institutions in the US.

However, such tools rapidly become unwieldy in large systems where the the dimensionality of the models concerned renders their detailed interpretation largely infeasible. Little research has yet been dedicated to the construction of summary measures designed to capture the salient features of such analyses at an appropriate level of aggregation. To this end, we propose a family of Generalised Connectedness Measures (GCMs) which extend the multi-market univariate or the single-market multivariate connectedness measures proposed by Diebold and Yilmaz (2009, 2011) to the truly multi-country and the multi-variable context.

Our GCMs are designed to facilitate the analysis and presentation of the inter-linkages between entities in large scale models at an appropriate and user-defined level of aggregation. To demonstrate its applicability, we apply our technique to evaluate the connectedness of the global economy using the macro-financial GVAR model developed by Greenwood-Nimmo et al. (2012a). This model is ideally suited to our purpose, comprising of a 176 variable system covering 26 countries that collectively account for the large majority of world trade. Although international linkages have been evaluated in similar models in the literature (c.f. DdPS), their treatment is necessarily highly selective, and this very selectivity introduces an ad hoc element into the analysis. However, it follows that conducting a comprehensive evaluation of the international and inter-variable linkages in such a model using the standard tools of dynamic analysis is all but impossible.[1] Therefore, although it is well-established that the GVAR framework largely circumvents the traditional curse of dimensionality associated with the limits imposed by the range and frequency of existing macroeconomic datasets, it introduces in turn a secondary curse of dimensionality whereby the limits of an individual's ability to process and interpret reported statistics become the binding constraint. This may be termed a processing constraint.

By computing GCMs at the level of individual variables (V-GCMs) or variable groups (G-GCMs), at the country-level (C-GCMs) and at the regional level (R-GCMs), we are able

[1]To satisfy oneself of the validity of this assertion, it is sufficient to note that it would be necessary to present $176^2 = 30,976$ separate impulse response functions to reveal the time-path of the effect of every possible individual shock on each variable in the system.

to map both the relative strength and direction of the connections in the global economy in a parsimonious and readily interpretable manner. In this way, we are able to circumvent the processing constraint and thereby unlock more of the potential of existing techniques for the estimation of large-scale economic models.

Our results reveal that...tbc. These findings are generally plausible and illuminating, and it is important to note that they have been obtained on the basis order-invariant GFEVDs derived from a simple reduced-form cointegrating GVAR model which has not been the subject of restrictions on either the long-run or contemporaneous matrices.

Our contribution to the literature is both timely and valuable. In the wake of the global financial crisis, there is an increasingly widespread and urgent desire to model the channels by which shocks are transmitted through the global economy. Furthermore, with the increasing popularity of the Panel and Global VAR frameworks, the number and range of models which may encounter processing constraints and would therefore benefit from the application of our GCMs is growing. Moreover, GCMs provide a rich source of information that may be used to supplement the range of preliminary data analyses and descriptive statistics that are currently reported in much applied empirical research. Indeed, GCMs can provide much richer insights than even the more sophisticated of the prevalent descriptive statistics such as time-varying correlation analysis. Our methodology therefore represents a valuable addition to the array of techniques that may be employed in exploratory data analysis.

Section 2 briefly outlines the current stance of the literature review related to systemic risks and macro-prudential policy. Section 3 introduces the structure of the GVAR model and describes the framework for dynamic interlinkage analysis. Section 4 extends the approach by Diebold and Yilmaz (2009, 2011) and develops a family of GCMs, through which we can construct several connectedness measures across countries, regions, variables, and any other relevant grouping. Section 5 presents main empirical findings by employing the same GNS model for the group of 33 countries (26 regions) over the extended sample period 1980Q2-2007Q2. Section 6 concludes.

2 Literature Overview

Given the increasing globalization and integration of economies and financial markets, especially the recent experience of a contagious global financial crisis, there is an increasingly widespread and urgent desire to explicitly model the complex interlinkages and interdependencies among entities with a system and the channels with which shocks are transmitted through the system. The aim is to improve policy analysis and risk management at multi-entity and supra-entity levels. To achieve this task, there are two issues to overcome: modelling issue and measurement issue. The former relates to how to account for the interlinkages and interdependencies in a large system without falling victim to the curse of dimensionality. The latter involves how to measure the connectedness among units within the system which could help measure and identify systemic risk.

Since using traditional VAR set-up for a large system warrants dimensional problems, several modelling approaches have been developed to address the first issue. Among all, the popular ones are Panel and Global Vector Autoregressive (VAR), and multi-country Dynamic Stochastic General Equilibrium (DSGE) frameworks. Each of these approaches has its own strengths but as the system becomes bigger and bigger, they all face with the same problem of 'processing constraint'.

Firstly, panel VAR models have been applied in multi-coutry analysis, good examples including Gavin and Theodorou (2005), Anderson, Qian, and Rasche (2006), Goodhart and Hofmann (2008), and Assenmacher-Wesche and Gerlach (2008). The use of panel VAR models has been mainly limited to analyse and test the economic relationship among variables of interest. For example, Goodhart and Hofmann (2008) analyse the linkages between money, credit, house prices and economic activity in a sample of 17 industrialized countries between 1970-2006 using a standard panel VAR set-up. By means of impulse response analysis, they find evidence of a significant multidirectional link between house prices, money, credit and the macroeconomy.

As Gavin and Theodorou (2005) and Goodhart and Hofmann (2008) point out, the advantage of panel VAR is that it increases the efficiency and power of the analysis, uncovers common dynamic relationships which might otherwise be obscured by the idiosyncratic effects at the country-specific level. However, the imposed pooling restrictions in the panel set-up disregard country-specific dynamics and cross-country interdependencies. This disadvantage of the standard panel VAR can be overcome though if one follows the re-parameterization/factorization approach of Canova et al. (2007) for multi-county VAR modelling, as demonstrated by Assenmacher-Wesche and Gerlach (2008). The number of parameters is reduced through different linear combinations (different factors) of regressors. Since the choice of factorization is application and possibly ad hoc, making the analysis based on a panel or multi-country VAR model inferior to that based on a GVAR model.

The GVAR framework originally developed by Pesaran, Schuermann and Weiner (2004, PSW), Dees, di Mauro, Pesaran and Smith (2007, DdPS) and Dees, Holly, Pesaran and Smith (2007, DHPS) offers a new approach to large scale macroeconometric modelling that circumvents the dimensional problems. The key innovation of GVAR is the exploitation of an underlying linking scheme, based on bilateral trading relations, to combine country-specific VAR models into a global system. This is achieved by the inclusion of weakly exogenous foreign variables in each country-specific model. These foreign variables are defined as trade-weighted averages of variables from all other countries. The use of trade-weighted foreign variables not only reduces the number of parameters in each country-specific VAR model, but also constitutes natural linkages among countries in the global system, making GVAR models more intuitive and coherent than panel or multi-country VAR models.

By virtue of their ability to explicitly model national, regional and global linkages, GVAR models represent a powerful tool for the analysis of global phenomena, including business cycle linkages (e.g. DdPS; DHPS), financial contagion (e.g. PSW; Chen et al., 2009; Sgherri and Galesi, 2009) and global imbalances (e.g. Bussière et al., 2009; Greenwood-Nimmo, Nguyen and Shin, 2012a, GNS; Greenwood-Nimmo, Nguyen and Shin, 2012b). For example, DdPS use a large system of 31 countries with long-term, short-term interest rates, inflation, output, exchange rate, equity and oil prices to analyze business cycle linkages between the US and the euro area. There impulse response analysis show that financial shocks are transmitted more rapidly than real shocks and financial markets have a higher degree of synchronization than real markets.

In contrast to the ease of estimation and empirical strength of GVAR models, calibrated

DSGE models offer rigorous theoretical microfoundations.[2] A number of two-country and multi-country DSGE models have emergence such as de Walque et al. (2005), Cristadoro et al. (2006) and the IMF's Global Economy Model (GEM), Global Fiscal Model (GFM) and Global Projection Model (GPM) which are neatly summarised by Bayoumi (2004), Botman et al. (2007) and Carabenciov et al. (2009). Nevertheless, large scale multi-country DSGE models remain relative rare due to the complexity of the modelling that is required to deliver the rich microfoundations that are considered the principle advantage of DSGE models relative to more data-driven approaches such as VAR.

Though differing in the modelling approaches, the above-mentioned studies all employ the standard tools of impulse response function (IRF) or forecast error variance decomposition (FEVD) or a combination of both to analyze the interlinkages among countries. However, large systems render detailed analysis infeasible, leading to selective presentation and interpretation of results. This selectivity may obscure a broad picture of interlinkages and hence systemic risk within the global system. Also, the lack of summary measures of connectedness makes it even harder to evaluate systemic risk properly in large scale model.

In the aftermath of the recent devastating global financial crisis, the IMF have provided a comprehensive report documenting popular methods to measure systemic linkages among financial institutions (Global Financial Stability Report, April 2009). The methods used to measure systemic linkages are: (i) the network approach, (ii) the co-risk model, (iii) the distress dependence matrix, and (iv) the default insensitive model. These methods provide alternative summary measures of the linkages to address systemic risk. Diverse and useful as they are, these methods only focus on the financial sector without explicitly accounting for the interactions between financial sector and the macroeconomy and the implications of systemic risk on the macroeconomy. These are important issues for economic management and stabilization policy. It is not clear how these shortcomings can be resolved within the set-ups of these methods.

In a similar direction, Diebold and Yilmaz (2009) provide a simple and intuitive measure of interdependence or spillovers of asset returns or volatilities by employing univariate vector autoregressive (VAR) models and aggregating orthogonalised (forecast error) variance decompositions across markets. In an analysis of 19 global equity markets over the period January 1992-November 2007, they find striking evidence of divergent dynamics that return spillovers display a gently increasing trend but no bursts, whereas volatility spillovers display no trend but clear bursts. However, results based on Cholesky-decomposition are well-established to be sensitive to ordering, as Cholesky-factor exact identification amounts to assumption of a particular recursive ordering, and cannot be tested empirically.

In this regard, Diebold and Yilmaz (2012) propose to consider a more general approach by employing the generalized forecast error variance decomposition developed by Pesaran and Shin (1998). They provide various intuitive directional and non-directional measures of connectedness to assess the interlinkages among financial institutions. However, tt is well-known that standard VAR models would suffer from dimensional problems as the system becomes bigger and bigger. Hence, a combination of multi-country multi-variate GVAR and connectedness measures suggested Diebold and Yilmaz (2012) will be a fruitful avenue

[2] A number of interesting intermediate cases obtain between the extremes of unrestricted VAR and DSGE, including over-identified cointegrating VAR and DSGE-VAR (c.f. Del Negro and Schorfheide, 2004).

to address the interlinkages and systemic risk in a system consisting of N entities and k variables for each entity.

3 Dynamic Linkages and Connectedness in The GVAR Model

The global VAR model comprises N economies indexed by $i = 1, .., N$. For each country-specific model, we denote the set of domestic variables by an $m_i \times 1$ vector, $\boldsymbol{y}_{it}$ and the associated country-specific foreign variables by an $m_i^* \times 1$ vector, $\boldsymbol{y}_{it}^*$ defined as $\boldsymbol{y}_{it}^* = \sum_{j=1}^{N} w_{ij} y_{it}$, where $w_{ij} \geq 0$ are the weights with $\sum_{j=1}^{N} w_{ij} = 1$, and $w_{ii} = 0$.[3]

We consider the following country-specific VARX* (p, p) model as[4]

$$\boldsymbol{y}_{it} = \sum_{j=1}^{p} \boldsymbol{\Phi}_{ij} \boldsymbol{y}_{i,t-j} + \sum_{j=0}^{p} \boldsymbol{\Phi}_{ij}^* \boldsymbol{y}_{i,t-j}^* + \boldsymbol{u}_{it} \tag{1}$$

where $\boldsymbol{\Phi}_{ij}$ and $\boldsymbol{\Phi}_{ij}^*$ are, respectively, $m_i \times m_i$ and $m_i \times m_i^*$ coefficient matrices, and $\boldsymbol{u}_{it} \sim iid\,(0, \boldsymbol{\Sigma}_{u,ii})$ with $\boldsymbol{\Sigma}_{u,ii}$ being an $m_i \times m_i$ positive definite variance–covariance matrix. Defining $\boldsymbol{z}_{it} = (\boldsymbol{y}_{it}, \boldsymbol{y}_{it}^*)'$, it follows that (1) can be written more compactly as

$$\boldsymbol{A}_{i0} \boldsymbol{z}_{it} = \sum_{j=1}^{p} \boldsymbol{A}_{ij} \boldsymbol{z}_{i,t-j} + \boldsymbol{u}_{it} \tag{2}$$

where $\boldsymbol{A}_{i0} = (\boldsymbol{I}_{m_i}, -\boldsymbol{\Phi}_{i0}^*)$, and $\boldsymbol{A}_{ij} = \left(\boldsymbol{\Phi}_{ij}, \boldsymbol{\Phi}_{ij}^*\right)$ for $j = 1, ..., p$.

The first step in constructing the GVAR model is to collect the $m \times 1$ vector of the global variables where $m = \sum_{i=1}^{N} m_i$,

$$\boldsymbol{y}_t = (\boldsymbol{y}_{1t}', ..., \boldsymbol{y}_{Nt}')'$$

Next, we define the $(m_i + m_i^*) \times m$ link matrices, $\boldsymbol{W}_i$'s, that are constructed using trade-weights, the financial-weights on the basis of the stock market capitalisations, the carefully selected spatial matrices or the combination of each.[5] It is then easily seen that the $\boldsymbol{z}_{it}$'s for each country-specific model can be expressed as

$$\boldsymbol{z}_{it} = \boldsymbol{W}_i \boldsymbol{y}_t$$

It is straightforward to express (2) in stacked form as

$$\boldsymbol{H}_0 \boldsymbol{y}_t = \sum_{j=1}^{p} \boldsymbol{H}_j \boldsymbol{y}_{t-j} + \boldsymbol{u}_t \tag{3}$$

[3]The definition of the weakly exogenous foreign variables for country i as weighted averages for countries/regions $i \neq j$ results in a simultaneous system of regional equations that may be solved to form a global system. It is also conventional to assign the country index 1 to the reference country, usually the US. This implies that the exchange rate of the reference country is determined in the remaining N country-specific models representing the rest-of-the-world (ROW). See DdPS and GNS for more details.

[4]For simplicity we do not include the deterministic regressors. But it is straightforward to include constant, trend and/or dummy shift variables, see Shin (2009) and GNS for details. Also the higher VAR order extension is straightforward.

[5]See for example DdPS and GNS for the detailed construction of the link matrices on the basis of trade-weights retrieved from the IMF's DOTS database.

where

$$\boldsymbol{H}_j = \begin{pmatrix} \boldsymbol{A}_{1j}\boldsymbol{W}_1 \\ \vdots \\ \boldsymbol{A}_{Nj}\boldsymbol{W}_N \end{pmatrix},\ j = 0, 1, ..., p;\ \boldsymbol{u}_t = \begin{pmatrix} \boldsymbol{u}_{1t} \\ \vdots \\ \boldsymbol{u}_{Nt} \end{pmatrix}$$

Finally, we obtain the reduced-form GVAR model by pre-multiplying (3) by $\boldsymbol{H}_0^{-1}$ as follows:

$$\boldsymbol{y}_t = \sum_{j=1}^{p} \boldsymbol{G}_j \boldsymbol{y}_{t-j} + \boldsymbol{\varepsilon}_t \tag{4}$$

where $\boldsymbol{G}_j = \boldsymbol{H}_0^{-1}\boldsymbol{H}_j$, $j = 1, ..., p$, are an $m \times m$ matrix of GVAR coefficients, and $\boldsymbol{\varepsilon}_t = \boldsymbol{H}_0^{-1}\boldsymbol{u}_t$.

Notice that all the GVAR parameters, $\boldsymbol{G}_j$ are obtained from the corresponding CVAR models and the transformation using the link matrices, $\boldsymbol{W}_i$ as given above. Global interactions now take place through three distinct but interrelated channels: First, there is direct dependence of country/region-specific variables, $\boldsymbol{y}_{it}$ on the corresponding foreign variables, $\boldsymbol{y}_{it}^*$ and their lagged values. Second, $\boldsymbol{y}_{it}$'s also depend on common global variables such as the crude oil price.

Finally, we discuss how to construct the $m \times m$ covariance matrices of $\boldsymbol{u}_t$ ($\boldsymbol{\Sigma}_u$) and $\boldsymbol{\varepsilon}_t$ ($\boldsymbol{\Sigma}_\varepsilon$). Since $\boldsymbol{\Sigma}_\varepsilon = \boldsymbol{H}_0^{-1}\boldsymbol{\Sigma}_u\boldsymbol{H}_0^{-1\prime}$, we focus on $\boldsymbol{\Sigma}_u$. Remind that the country-specific $m_i \times m_i$ covariance matrix of $\boldsymbol{u}_{it}$, denoted $\boldsymbol{\Sigma}_{u,ii}$, can be consistently estimated from the corresponding CVAR model, which allows for non-zero contemporaneous correlations among the shocks to all $\boldsymbol{y}_{it}$. There are a couple of options for estimating contemporaneous cross-country covariances, $E\left(\boldsymbol{u}_{it}\boldsymbol{u}_{jt}'\right) = \boldsymbol{\Sigma}_{u,ij}$.[6] First, we may allow the shocks in $\boldsymbol{u}_{it}$ to be contemporaneously correlated across countries/regions, and estimate them nonparametrically. In this case we have:

$$\boldsymbol{\Sigma}_u = \begin{bmatrix} \boldsymbol{\Sigma}_{u,11} & \boldsymbol{\Sigma}_{u,12} & \cdots & \boldsymbol{\Sigma}_{u,1N} \\ \boldsymbol{\Sigma}_{u,21} & \boldsymbol{\Sigma}_{u,22} & \cdots & \boldsymbol{\Sigma}_{u,2N} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{\Sigma}_{u,N1} & \boldsymbol{\Sigma}_{u,N2} & \cdots & \boldsymbol{\Sigma}_{u,NN} \end{bmatrix} \tag{5}$$

It follows that countries can also be inter-linked via contemporaneous cross-country covariances, $\boldsymbol{\Sigma}_{u,ij}$. Considering that the dimension of the GVAR model is relatively large, however, we may argue that many of off-diagonal blocks in $\boldsymbol{\Sigma}_u$ are likely to be less precisely estimated or statistically insignificant, especially the blocks associated with the relatively small or developing countries/regions. Notice also that the direct impacts of the (weighted average of) foreign variables have already been incorporated in estimating the CAVR parameters including $\boldsymbol{\Sigma}_{u,ii}$. In this regard, we consider the second option by imposing the block diagonality in (5), and construct the more parsimonious global covariance matrix as follows:[7]

$$\boldsymbol{\Sigma}_u^d = \begin{bmatrix} \boldsymbol{\Sigma}_{u,11} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma}_{u,22} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \boldsymbol{\Sigma}_{u,NN} \end{bmatrix} \tag{6}$$

[6] Under the maintained assumption that the lag order of the CVARX model is sufficiently large, u_{it}'s are not serially correlated.

[7] **We may check weak CSD conditions under which the block-diagonality holds trivially.**

3.1 Forecast Error Variance Decomposition and Impulse Response Function

In order to derive the dynamic features of the GVAR model such as FEVD and IRF, we now express the GVAR model, (4) as the infinite order global vector moving average (GVMA) representation:

$$\boldsymbol{y}_t = \sum_{j=0}^{\infty} \boldsymbol{B}_j \boldsymbol{\varepsilon}_{t-j}, \tag{7}$$

where $\boldsymbol{B}_j$ are evaluated recursively as

$$\boldsymbol{B}_j = \boldsymbol{G}_1 \boldsymbol{B}_{j-1} + \boldsymbol{G}_2 \boldsymbol{B}_{j-2} + \cdots + \boldsymbol{G}_{p-1} \boldsymbol{B}_{j-p+1},\ j = 1, 2, ...,\ \text{with } \boldsymbol{B}_0 = \boldsymbol{I}_m,\ \boldsymbol{B}_j = \mathbf{0} \text{ for } j < 0.$$

Following Pesaran and Shin (1998), it is straightforward to derive the generalised and the orthogonalised FEVDs as follows:

$$GFEVD\left(y_{jt}; u_{it}, h\right) = \frac{\sigma_{u,ii}^{-1} \sum_{\ell=0}^{h-1} \left(\mathbf{e}_j' \boldsymbol{B}_\ell \boldsymbol{H}_0^{-1} \boldsymbol{\Sigma}_u \mathbf{e}_i\right)^2}{\sum_{\ell=0}^{h-1} \mathbf{e}_j' \boldsymbol{B}_\ell \boldsymbol{\Sigma}_\varepsilon \boldsymbol{B}_\ell' \mathbf{e}_j} \tag{8}$$

$$OFEVD\left(y_{jt}; u_{it}, h\right) = \frac{\sum_{\ell=0}^{h-1} \left(\mathbf{e}_j' \boldsymbol{B}_\ell \boldsymbol{H}_0^{-1} \boldsymbol{P} \mathbf{e}_i\right)^2}{\sum_{\ell=0}^{h-1} \mathbf{e}_j' \boldsymbol{B}_\ell \boldsymbol{\Sigma}_\varepsilon \boldsymbol{B}_\ell' \mathbf{e}_j} \tag{9}$$

for $i.j\ =, 1..., m$, where $h = 1, 2, ...$ is the forecast horizon, $\boldsymbol{\Sigma}_\varepsilon = \boldsymbol{H}_0^{-1} \boldsymbol{\Sigma}_u \boldsymbol{H}_0^{-1\prime}$, $\boldsymbol{P}$ is the lower triangular matrix obtained via the Choleski decomposition of $\boldsymbol{\Sigma}_u = \boldsymbol{P}\boldsymbol{P}'$. Similarly, the generalised and the orthogonalised IRFs are derived as follows:

$$GIRF_{ji,\ell} = \frac{\mathbf{e}_j' \boldsymbol{B}_\ell \boldsymbol{H}_0^{-1} \boldsymbol{\Sigma}_u \mathbf{e}_i}{\sqrt{\sigma_{ii}}},\ \ell = 0, 1, 2, ... \tag{10}$$

$$OIRF_{ji,\ell} = \mathbf{e}_j' \boldsymbol{B}_\ell \boldsymbol{H}_0^{-1} \boldsymbol{P} \mathbf{e}_i,\ \ell = 0, 1, 2, ... \tag{11}$$

Notice that the sum of OFEVDs is equal to 100%, but the sum of GEFVDS may exceed 100% due to the non-zero correlations across shocks. Hence, we follow Diebold and Yilmaz (2011) **(wang 2002??)** and develop the "normalized" $GFEVD$ as follows: Let

$$g_{ji} = GFEVD\left(y_{jt}; u_{it}, h\right) \tag{12}$$

be the variance share of y_{jt} in the generalized forecast error variance of y_{it}, then define the normalized $GFEVD$ by

$$\eta_{ji} = \frac{g_{ji}}{\sum_{i=1}^{m} g_{ji}} \tag{13}$$

It is easily seen that

$$\sum_{i=1}^{m} \eta_{ji} = 1 \text{ and } \sum_{j=1}^{m} \left(\sum_{i=1}^{m} \eta_{ji} \right) = m. \tag{14}$$

In this scheme, the total sum of the forecast error variance shares of each variable is normalized to 100%, which may be more robust to the presence of unusually large forecast error variances of some variables. *[Viet: more intuitive details using the previous exercises??]*

Some discussions on FEVD-based connectedness and IRF-based shock propagation mechanism here??

4 Generalised Connectedness Measues in the GVAR Model

4.1 Connectedness among m variables

Now, it is straightforward to extend the Diebold and Yilmaz's (2011) approach into the GVAR framework with multiple variables and multiple countries. Using the normalized GFEVDs, we can construct the $m \times m$ matrix of connectedness table for the $m \times 1$ vector of global variables by[8]

$$\mathbb{C} = \begin{bmatrix} \eta_{1,1} & \cdots & \eta_{1,m_1} & \eta_{1,m_1+1} & \cdots & \eta_{1,m_1+m_2} & \cdots & \eta_{1,m} \\ \vdots & \ddots & \vdots & \vdots & & \vdots & & \vdots \\ \eta_{m_1,1} & \cdots & \eta_{m_1,m_1} & \eta_{m_1,m_1+1} & \cdots & \eta_{m_1,m_1+m_2} & \cdots & \eta_{m_1,m} \\ \eta_{m_1+1,1} & \cdots & \eta_{m_1+1,m_1} & \eta_{m_1+1,m_1+1} & \cdots & \eta_{m_1+1,m_1+m_2} & & \eta_{m_1+1,m} \\ \vdots & & \vdots & \vdots & \ddots & \vdots & & \vdots \\ \eta_{m_1+m_2,1} & \cdots & \eta_{m_1+m_2,m_1} & \eta_{m_1+m_2,m_1+1} & \cdots & \eta_{m_1+m_2,m_1+m_2} & & \eta_{m_1+m_2,m} \\ \vdots & & & & & & \ddots & \vdots \\ \eta_{m,1} & \cdots & \eta_{m,m_1} & \eta_{m,m_1+1} & \cdots & \eta_{m,m_1+m_2} & \cdots & \eta_{m,m} \end{bmatrix} \tag{15}$$

We now define own-variable variance share (H_j) and total cross-variable variance share (F_j) as follows:

$$H_j = \eta_{j,j};\ F_j = \sum_{i=1, i\neq j}^{m} \eta_{j,i};\ O_j + F_j = 1 \tag{16}$$

Notice that F_j measures the total "from" contributions of all other variables to y_{jt} (the total directional connectedness from others to y_{jt}), as the fractions of the h-step-ahead error variances in forecasting y_{jt} due to shocks to all y_{it}'s for $i = 1, ..., m$ and $i \neq j$.[9] Similarly, we can define the total "to" contributions of of y_{jt} to all other variables as the column sum minus its own contribution; namely

$$T_j = \sum_{i=1, i\neq j}^{m} \eta_{i,j} \tag{17}$$

which measures the total directional connectedness from y_{jt} to others. Then, the net effect is defined by

$$N_j = T_j - F_j \tag{18}$$

which measures the net directional connectedness of y_{jt}.

It is now straightforward to develop the following aggregate (non-directional) measures for the $m \times 1$ vector of global variables:

$$H = \frac{1}{m} \sum_{j=1}^{m} H_j \tag{19}$$

[8] Notice that the sum of each row is normalised to 100%.

[9] Diebold and Yilmaz (2009) denote this measure as the spillover index in the context of the single returns or volatilities across the multiple stock markets.

$$S=\frac{1}{m}\sum_{j=1}^{m}F_j=\frac{1}{m}\sum_{j=1}^{m}T_j \tag{20}$$

We call H and S the aggregate heatwave index and the aggregate cross variance shares (spillovers), respectively, following the broad tradition of Engle et al. (1990) and Diebold and Yilmaz (2009). Notice by construction that

$$H+S=1 \quad \text{and} \quad \sum_{j=1}^{m}N_j=0$$

The second equality implies that the total net connectedness among all the m variables is simply equal to zero.

Notice that F_j and T_j can be further decomposed into the total domestic and foreign contributions as follows:

$$F_j=F_j^W+F_j^B \text{ and } T_j=T_j^W+T_j^B$$

where F_j^W $\left(T_j^W\right)$ measures the total from (to) contributions of all other within-country variables to y_{jt}, and F_j^B $\left(T_j^W\right)$ measures the total from (to) contributions of all other between-country variables to y_{jt}. Similarly, we can define the aggregate directional measures as

$$F^W=\frac{1}{m}\sum_{j=1}^{m}F_j^W,\ F^B=\frac{1}{m}\sum_{j=1}^{m}F_j^B,\ T^W=\frac{1}{m}\sum_{j=1}^{m}T_j^W,\ T^B=\frac{1}{m}\sum_{j=1}^{m}T_j^B \tag{21}$$

where F^W $\left(T^W\right)$ measures the intra-counrty aggregate from (to) contributions, and F^B $\left(T^W\right)$ measures the inter-country aggregate from (to) contributions. Unlike the undirectional aggregate spillover measure, S, careful investigation of the directional aggregate measures such as F^W, T^W, F^B and T^W, will shed further lights on the global connectedness across different forecast horizons and across different time periods.

4.2 Connectedness across N countries or R regions

Now, we show how to transform the variable-connectedness table in (15) into the country-connectedness table. By selecting the associated country blocks from the variable-connectedness table in (15), we can express $\mathbb{C}$ as

$$\mathbb{C}=\begin{bmatrix}\mathbf{C}_{1,1} & \mathbf{C}_{1,2} & \cdots & \mathbf{C}_{1,N}\\ \mathbf{C}_{2,1} & \mathbf{C}_{2,2} & \cdots & \mathbf{C}_{2,N}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{C}_{N,1} & \mathbf{C}_{N,2} & \cdots & \mathbf{C}_{N,N}\end{bmatrix} \tag{22}$$

where $\mathbf{C}_{k,\ell}$ is the $m_k\times m_\ell$ matrix of the cross-country block given by

$$\mathbf{C}_{k,\ell}=\begin{bmatrix}\eta_{\tilde{m}_k+1,\tilde{m}_\ell+1} & \cdots & \eta_{\tilde{m}_k+1,\tilde{m}_\ell+m_\ell}\\ \vdots & \ddots & \vdots\\ \eta_{\tilde{m}_k+m_k,\tilde{m}_\ell+1} & \cdots & \eta_{\tilde{m}_k+m_k,\tilde{m}_\ell+m_\ell}\end{bmatrix},\quad k,\ell=1,...,N \tag{23}$$

with $\tilde{m}_h = \sum_{k=1}^{h-1} m_k$ and m_k being the number of variables in country k. Notice that $\mathbf{C}_{k,k}$ contains the total intra-country variance contributions within country k while $\mathbf{C}_{k,\ell}$ for $k \neq \ell$ contains the total intra-country variance contributions from country ℓ to country k.

Using the country-connectedness table in (22), we can define the following connectedness measures across N countries. First, we define the total intra-country variance contribution in country k by

$$O_k^C = \mathbf{1}_k' \mathbf{C}_{k,k} \mathbf{1}_k \tag{24}$$

where $\mathbf{1}_k$ is $m_k \times 1$ column vector of ones. Next, the total inter-country variance contribution from country ℓ to country k is similarly defined as

$$F_{k,\ell}^C = \mathbf{1}_k' \mathbf{C}_{k,\ell} \mathbf{1}_\ell \tag{25}$$

Furthermore, the total intra-country variance contribution, O_k^C, can be decomposed into own-variable and cross-variable variance shares within country k such that

$$O_k^C = O_k^{CO} + O_k^{CC} \tag{26}$$

where the total intra-country own-variable variance share is given by

$$O_k^{CO} = trace\left(\mathbf{C}_{k,k}\right) \tag{27}$$

and the total intra-country cross-variable variance share is simply obtained as

$$O_k^{CC} = O_k^C - O_k^{CO}. \tag{28}$$

Based on these country-based connectedness measures, we now obtain the following $N \times N$ matrix of country-connectedness table:[10]

$$\mathbb{C}_C = \begin{bmatrix} O_1^C & F_{1,2}^C & \cdots & F_{1,N}^C \\ F_{2,1}^C & O_2^C & \cdots & F_{2,N}^C \\ \vdots & \vdots & \ddots & \vdots \\ F_{N,1}^C & F_{N,2}^C & \cdots & O_N^C \end{bmatrix} \tag{29}$$

Using the country-connectedness table in (29), we can develop the aggregate country connectedness measures as follows: the total "from", "to" and "net" contributions for country k are defined as follows:

$$F_k^C = \sum_{\ell=1,\ell\neq k}^{N} F_{k,\ell}^C, \quad T_k^C = \sum_{\ell=1,\ell\neq k}^{N} F_{\ell,k}^C, \quad N_k^C = T_k^C - F_k^C \tag{30}$$

where F_k^C measures the total inter-country variance contributions from all other countries to country k (total intra-country from contribution), T_k^C measures the total inter-country variance contributions from country k to all other countries (total intra-country to contribution), and N_k^C is the net contribution associated with country k.

[10]Notice now that the sum of the country k is now equal to $m_k \times 100\%$.

Next, we derive the following "aggregate" country connectedness measures for N countries:

$$H^C = \frac{1}{mN} \sum_{k=1}^{N} O_k^C = \frac{1}{mN} \sum_{k=1}^{N} \left(O_k^{CO} + O_k^{CC} \right) \tag{31}$$

$$S^C = \frac{1}{mN} \sum_{k=1}^{N} F_k^C = \frac{1}{mN} \sum_{k=1}^{N} T_k^C \tag{32}$$

where H^C and S^C are the aggregate Heatwave and Spillover Indexes in terms of N countries, respectively. Once again by construction,

$$H^C + S^C = 1 \quad \text{and} \quad \sum_{k=1}^{N} N_k^C = 0. \tag{33}$$

We can derive regional connectedness measures in a similar fashion to the country case and express $\mathbf{C}$ in (15) as

$$\mathbb{C} = \begin{bmatrix} \mathbf{R}_{1,1} & \mathbf{R}_{1,2} & \cdots & \mathbf{R}_{1,R} \\ \mathbf{R}_{2,1} & \mathbf{R}_{2,2} & \cdots & \mathbf{R}_{2,R} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{R}_{R,1} & \mathbf{R}_{R,2} & \cdots & \mathbf{R}_{R,R} \end{bmatrix} \tag{34}$$

where $\mathbf{R}_{r,s}$ is the $m_r \times m_s$ matrix of the regional block. Notice that $\mathbf{R}_{r,r}$ contains the total intra-region variance contributions within region r while $\mathbf{R}_{r,s}$ contains the total intra-region variance contributions from region s to region r. Then, using a similar logic, we can obtain the $R \times R$ matrix of regional connectedness table as follows:

$$\mathbb{C}_R = \begin{bmatrix} O_1^R & F_{1,2}^R & \cdots & F_{1,R}^R \\ F_{2,1}^R & O_2^R & \cdots & F_{2,R}^R \\ \vdots & \vdots & \ddots & \vdots \\ F_{R,1}^R & F_{R,2}^R & \cdots & O_R^R \end{bmatrix} \tag{35}$$

Using the regional connectedness table in (35), we can define the aggregate regional connectedness measures associated with region r by

$$F_r^R = \sum_{r=1, r\neq s}^{R} F_{r,s}^R, \quad T_r^R = \sum_{r=1, r\neq s}^{R} F_{s,r}^R, \quad N_r^R = T_r^R - F_r^R \tag{36}$$

where F_r^R measures the total variance contributions from all other regions to region r (the total inter-regional from contribution), T_r^R measures the total variance contributions from region r to all other regions (total intra-regional to contribution), and N_r^R is the net contribution associated with region r. Next, we derive the "aggregate" regional connectedness measures:

$$H^R = \frac{1}{mR} \sum_{r=1}^{R} O_r^R = \frac{1}{mR} \sum_{r=1}^{R} \left(O_r^{RO} + O_r^{RC} \right) \tag{37}$$

$$S^R = \frac{1}{mR}\sum_{r=1}^{R} F_r^R = \frac{1}{mR}\sum_{r=1}^{R} T_r^R \tag{38}$$

where H^R and S^R are the aggregate Heatwave and Spillover Indexes in terms of R regions, respectively. Once again by construction,

$$H^R + S^R = 1 \quad \text{and} \quad \sum_{r=1}^{R} N_r^R = 0. \tag{39}$$

Remark: In the case we are interested in the individual country's connectedness with the regions, we simply set the first region to the country of interest. Then, we can easily examine the total connectedness measures associated with regions as follows:

$$F_1^R = \sum_{s=2}^{R} F_{1,s}^R, \quad T_1^R = \sum_{s=2}^{R} F_{s,1}^R, \quad N_1^R = T_1^R - F_1^R \tag{40}$$

where F_1^R measures the total inter-regional variance contributions from all other regions to the country of interest, T_1^R measures the total inter-regional variance contributions from the country of interest to all other regions, and N_1^R is the net contribution associated with the country of interest.

4.3 Connectedness among G variable-groups

We may be interested in measuring connectedness in terms of the group of variables, e.g. financial variables vs variables and order flows and prices. We now derive the common-variable connectedness measures in a similar fashion to the country or the regional case. We now regroup the table for $m \times 1$ vector of global variables into the table for G variables. Then, we can express $\mathbf{C}$ in (15) as

$$\mathbb{C} = \begin{bmatrix} \mathbf{G}_{1,1} & \mathbf{G}_{1,2} & \cdots & \mathbf{G}_{1,G} \\ \mathbf{G}_{2,1} & \mathbf{G}_{2,2} & \cdots & \mathbf{G}_{2,G} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{G}_{G,1} & \mathbf{G}_{G,2} & \cdots & \mathbf{G}_{G,G} \end{bmatrix} \tag{41}$$

where $\mathbf{G}_{g,h}$ is the $n_g \times n_h$ matrix of the variable block with n_g being the number of countries in the variable group, g. Notice that $\mathbf{G}_{g,g}$ contains the total common-variable variance contributions while $\mathbf{G}_{g,h}$ contains the total cross-variable variance contributions from variable h to variable g.

We now define the common-variable connectedness measures as follows: First, the total common-variable variance contribution for variable g is given by

$$O_g^G = \mathbf{1}_g' \mathbf{G}_{g,g} \mathbf{1}_g \tag{42}$$

which is further decomposed into intra and inter-country variance shares such as

$$O_g^G = O_g^{GW} + O_g^{GB} \tag{43}$$

where the total intra own-variable variance share is given by

$$O_g^{GW} = trace\left(\mathbf{G}_{g,g}\right) \tag{44}$$

Next, the total cross-variable variance contribution from variable h to variable g is defined as

$$F_{g,h}^G = \mathbf{1}_g'\mathbf{G}_{g,h}\mathbf{1}_h \tag{45}$$

Notice now that $F_{g,h}^G$ is further decomposed into intra and inter-country variance shares such as

$$F_{g,h}^G = F_{g,h}^{GW} + F_{g,h}^{GB} \tag{46}$$

where the total intra cross-variable variance share is given by

$$F_{g,h}^{GW} = \frac{1}{n_g} trace\left(\mathbf{G}_{g,h}\right) \tag{47}$$

Then, we obtain the following $G \times G$ matrix of group-variable connectedness table:

$$\mathbb{C}_G = \begin{bmatrix} O_1^G & F_{1,2}^G & \cdots & F_{1,G}^G \\ F_{2,1}^G & O_2^G & \cdots & F_{2,G}^G \\ \vdots & \vdots & \ddots & \vdots \\ F_{G,1}^G & F_{G,2}^G & \cdots & O_G^G \end{bmatrix} \tag{48}$$

Using (48), we can define the aggregate group-variable connectedness measures as follows:

$$F_g^G = \sum_{h=1,h\neq g}^{G} F_{g,h}^G, \quad T_g^G = \sum_{h=1,h\neq g}^{G} F_{h,g}^G, \quad N_g^G = T_g^G - F_g^G \tag{49}$$

where F_g^G measures the total variance contributions from all other variables to variable g, T_g^G measures the total variance contributions from variable g to all other variables, and N_g^G is the net contribution associated with variable g.

Next, we derive the following "aggregate" group-variable connectedness measures:

$$H^G = \frac{1}{G}\sum_{g=1}^{G} O_g^G = \frac{1}{G}\sum_{g=1}^{G}\left(O_g^{GW} + O_g^{GB}\right) \tag{50}$$

$$S^G = \frac{1}{G}\sum_{g=1}^{G} F_g^G = \frac{1}{G}\sum_{g=1}^{G} T_g^G \tag{51}$$

where H^G and S^G are the aggregate Heatwave and Spillover Indexes in terms of G variables, respectively. Once again by construction,

$$H^G + S^G = 1 \quad \text{and} \quad \sum_{g=1}^{G} N_g^G = 0. \tag{52}$$

Remark: Suppose that we consider the two further decompositions in (43) and (46). In this case we can express $\mathbb{C}_G$ in (48) as

$$\begin{aligned}\mathbb{C}_G &= \mathbb{C}_G^W + \mathbb{C}_G^B \\ &= \begin{bmatrix} O_1^{GW} & F_{1,2}^{GW} & \cdots & F_{1,G}^{GW} \\ F_{2,1}^{GW} & O_2^{GW} & \cdots & F_{2,G}^{GW} \\ \vdots & \vdots & \ddots & \vdots \\ F_{G,1}^{GW} & F_{G,2}^{GW} & \cdots & O_N^{GW} \end{bmatrix} + \begin{bmatrix} O_1^{GB} & F_{1,2}^{GB} & \cdots & F_{1,G}^{GB} \\ F_{2,1}^{GB} & O_2^{GB} & \cdots & F_{2,G}^{GB} \\ \vdots & \vdots & \ddots & \vdots \\ F_{G,1}^{GB} & F_{G,2}^{GB} & \cdots & O_N^{GB} \end{bmatrix}\end{aligned} \tag{53}$$

Using (53), we now define the following aggregate group-variable connectedness measures:

$$F_g^{GW} = \sum_{h=1,h\neq g}^{G} F_{g,h}^{GW}, \quad T_g^{GW} = \sum_{h=1,h\neq g}^{G} F_{h,g}^{GW}, \quad N_g^{GW} = T_g^{GW} - F_g^{GW} \tag{54}$$

$$F_g^{GB} = \sum_{h=1,h\neq g}^{G} F_{g,h}^{GB}, \quad T_g^{GB} = \sum_{h=1,h\neq g}^{G} F_{h,g}^{GB}, \quad N_g^{GB} = T_g^{GB} - F_g^{GB} \tag{55}$$

where F_g^{GW} $\left(F_g^{GB}\right)$ measures the total intra (inter) country variance contributions from all other variables to variable g, T_g^{GW} $\left(T_g^{GB}\right)$ measures the total intra (inter) country variance contributions from variable g to all other variables, and N_g^{G} $\left(N_g^{GB}\right)$ is the corresponding net contribution. In particular, the normalised GFEVDs for variable g, can be decomposed into four components,

$$O_g^{GW} + O_g^{GB} + F_g^{GW} + F_g^{GB} = 100\% \tag{56}$$

where O_g^{GW} $\left(O_g^{GB}\right)$ measures the total intra (inter) country own-variable variance contributions.

Remark: Suppose that we are interested in connectedness of the single variable of individual country (or the common factor such as the oil price) with all other group variables. In this case we simply place that single variable in the first group variable block, $\mathbf{G}_{1,1}$ in (41) with $G-1$ group of variables.[11] In this case $\mathbf{G}_{1,1}$ is a scalar and $\mathbf{G}_{1,g}$ is the $1 \times n_g$ row vector. Then, we can easily examine the total connectedness measures as follows:

$$F_1^G = \sum_{h=2}^{G} F_{1,h}^G, \quad T_1^G = \sum_{h=2}^{G} F_{h,1}^G, \quad N_1^G = T_1^G - F_1^G \tag{57}$$

where F_1^G measures the total variance contributions from all other variables to the single variable of the country of interest, T_1^G measures the total variance contributions from the single variable of the country of interest to all other variables, and N_1^G is the net contribution associated with the single variable of the country of interest. Furthermore, we also consider two further decompositions similar to (43) and (46). But, notice that $O_1^G = O_1^{GW}$ since it is a scalar. Now, by construction, we obtain O_1^{GB} by

$$O_1^{GB} = F_{1,2}^G$$

[11] For the case with a single variable of the particular individual country, we place the same variables of other countries in the 2nd block.

Accordingly, $F^G_{1,g}$ can be decomposed into intra and inter-country variance shares such as

$$F^G_{1,h} = F^{GW}_{1,h} + F^{GB}_{1,h} \text{ for } h = 3, ..., G, \tag{58}$$

where $F^{GW}_{1,h}$ is the first element of the $1 \times n_h$ vector of $\mathbf{G}_{1,h}$. Hence, we can construct the associated total connectedness measures as follows:

$$F^{GW}_1 = \sum_{h=3}^{G} F^{GW}_{1,h}, \quad T^{GW}_1 = \sum_{h=3}^{G} F^{GW}_{h,1}, \quad N^{GW}_1 = T^{GW}_1 - F^{GW}_1 \tag{59}$$

$$F^{GB}_1 = \sum_{h=3}^{G} F^{GB}_{1,h}, \quad T^{GB}_1 = \sum_{h=3}^{G} F^{GB}_{h,1}, \quad N^{GB}_1 = T^{GB}_1 - F^{GB}_1 \tag{60}$$

Then, the normalised GFEVDs for the single variable of the country of interest, can also be decomposed into four components,

$$O^{GW}_1 + O^{GB}_1 + F^{GW}_1 + F^{GB}_1 = 100\% \tag{61}$$

5 Empirical Application

5.1 Generalized Forecast Error Variance Decompositions of 4 Focus Countries

In this Section, we seek to understand the international linkages of the four focus countries, i.e US, Eurozone, Japan and China by use of normalized forecast error variance decompositions (FEVDs). For simplicity of analysis, the normalized FEV is decomposed into four components as follows:

- *Own* denotes the proportion of the variable's FEV explained by the variable itself,
- *O. domestic* the proportion explained by the remaining domestic variables,
- *Oil* the proportion explained by the oil price,
- *Foreign* the proportion explained by all foreign variables excluding oil price.

Note that in the global model, the US model only has six domestic variables (excluding the oil price) and the US price level is included instead of the US inflation while China model does not include a stock market index.

At a glance, several stylized facts emerge from the FEVDs of the four focus countries. Firstly, the influence of the oil price is relatively mute in the US, Eurozone and China but considerably large in Japan. Secondly, the own contribution typically dominates in the short horizon but its importance tends to decreases as the horizon increases, though the decreasing decree varies across countries and variables. Thirdly, in the US, Eurozone and China, the influence of domestic factors (combination of own and domestic contributions) dominates the influence of foreign factors (contributions of oil price and all foreign variables) at all horizons, with the only exception being the Eurozone stock market. Meanwhile, the overwhelming dominance of foreign factors in Japan, especially in the medium- to long-term, characterizes the trade-reliant nature of the Japanese economy.

Fourthly, as one might expect, foreign influence is larger in exports than in imports in all countries but the opposite is observed for domestic influence. Finally, while own countribution overwhelmingly dominates the influence from other sources in case of the US stock market, foreign influence, on the contrary, is the dominant force in the stock markets of the Eurozone and Japan. A closer analysis shows that the US stock market inovation contributes around 30% of the FEVDs of the Eurozone and Japan's stock markets across all horizons. This finding illustrates (i) the dominant role of the US stock market in the global stock market; (ii) the close linkages among major stock markets; and (iii) the relatively stronger financial tights among major economies than their "real" tights.

Overall, the analysis of FEVDs suggests that the US, China and, to a large extent, the Eurozone exhibit the behaviours of large closed economies while Japan is clearly a large open economy dependant on external trade. The limited influence of foreign factors on the US and China economies may have different explanations. The US is the largest and dominant economy in the world with domestic consumption accounting for roughly 60% of its GDP and hence the modest influence of foreign factors. The limited influence of foreign factors on China's economy, on the other hence, is due to both the dominance of domestic fixed investments (around 50% of its GDP for the last two decades) and the tight control of the government over many aspects of the economy, e.g. exchange rate, capital flow...

5.1.1 US

Figure 1 shows that, firstly, the contribution of foreign factors (oil price and all foreign variables) is relatively small in the US. The largest contribution of foreign factors is recorded in the US imports, exports and price level, as one would expect, and is around 20-30%. The influence of oil price is only material to the US price level (contributing around 20%) mainly in the short- to medium-term, and to the US real output mainly in the medium- to long-term (contributing around 10%). Secondly, the FEVD of the US interest rate suggests that the monetary policy is conducted mainly based on domestic conditions as the contribution from other domestic variables accounts for more than 60% from the second quarter. Thirdly, the contribution from other domestic variables also dominates the FEVDs of the US imports and exports from the second quarter on, accounting for roughly 60% and 50% respectively. A noticable difference is the FEVDs of exports and imports is that while own contribution decreases considerably in imports, ending up at less than 10% in the long-term, own contribution remains relatively stable at around 20% in exports from the medium- to long-term.

[Insert Figure 1 about here]

Next, as dicussed above, own contribution is overwhelmingly dominant in the US stock market while foreign contribution gradually increases to around 20% in the long-term. Fifthly, the FEVD of the US price level is dominated by own contribution and the contribution from the oil price in the short-term by by the contrubtions of other domestic and foreign variables from the medium- to long-term. Finally, turning to the FEVD of real output, domestic factors account for roughly 90% in the short-term and about 70% in the long-term, which is plausible considering that domestic consumption is the largest component of the US GDP.

5.1.2 EU

The FEVDs of interest rate, imports, exports and real output in the Eurozone offer similar stories to those of the US with the dominant influence of domestic factors. However, somewhat different patterns of FEVDs are observed for the Eurozone stock market and inflation. Specifically, the Eurozone stock market is largely determined by foreign factors which account for around 60% of its FEV. As for inflation, own contribution accounts for 50% of it FEV at the first quarter but drops to just 5% in the second quarter, giving rise to the contributions of other domestic and foreign variables. Apparently, the oil price has little influence on the Eurozone inflation. Finally, the FEVD of the Eurozone real exchange rate suggests that the exchange rate is mainly determined by domestic factors (as opposed to the Korean case).

[Insert Figure 2 about here]

5.1.3 Japan

The FEVDs of Japan's variables are quite different from those from the US, Eurozone and China, probably reflecting the structural difference between Japan's economy and thoses of the other focus countries. As summarized earlier, the contributions of foreign factors dominate those of domestic factors from the medium- to long-term. The influence of domestic factors is only considerable in the short-term in most cases. The largest own contribution is observed in interest rate and real output while the largest contribution from other domestic variables is reported for real exchange rate. The large contribution of the oil price in the FEVDs of imports clearly indicates the importance of oil, and resource in general, imports to Japanese manufacturing sector. Moreover, the large contribution of the oil price in interest rate suggests that monetary policy responses considerably to the fluctuation of the oil price, probably offsetting the negative impact of oil price fluctuations on the manufacturing sector.

Next, the oil price does exert impacts on all of Japanese variables but exports. The largest impacts of the oil price are recorded in interest rate and imports, followed by real exchange rate, stock market and inflation. The impact of the oil price on real output is only visible from the medium term. Finally, the FEVD of real output suggests that it is determined by domestic factors in the short- to medium-term but by foreign factors from the medium- to long-term.

[Insert Figure 3 about here]

The FEVDs of Japanese variables boast similarities to those of Korean variables, which reflects the smilarities in the structure of these two economies. In particular, both pursue the exports-led growth strategy and rely considerable on external trade for income, and hence the importance of foreign factors to their economies.

5.1.4 China

The FEVDs of Chinese variables offer similar stories to those of the US and Eurozone variables. Specifically, foreign factors have ver limited influence on Chinese variables. The largest foreign contribution is reported for real exchange rate, followed by inflation, exports, interest rate and imports. Own contribution often dominates in the short- to medium-term in many cases. The contribution from other domestic variables is dominant in real exchange rate, interest rate, imports and exports, especially from the medium- to long-term. Chinese real output largely depends on itself due to the sustained high level of fixed investments and government spending over the sample period.

[Insert Figure 4 about here]

The above analysis suggests that despite enormous trade links with other countries Chinese economy is still very much a closed one.

5.2 Connectedness Results

5.2.1 Some Important Notes

To avoid any misunderstanding or misinterpretation of the results, in this subsection I briefly explain the results that I provided in the figures - from Figure 5 to Figure 11. Specifically, we start from the 176-variable connectedness table (retrieved from GFEVDs of 176 variables) and further summarize into 26-country, 12-region, 8-variable-group connectedness tables.

First, the contributions in the different connectedness tables, as provided, can be grouped into *from* and *to* contributions.

Second, both *from* and *to* contributions can be further divided into *own*, *within* and *between* contributions. These are defined as followed,

- *own*: own-variable variance contribution,
- *within*: within-country/region/group cross-variable variance contribution,
- *between*: between-country/region/group cross-variable variance contribution.

Third, *from* contributions of each variable are normalized so that they sum to 100%, therefore the sum of *from-own*, *from-within* and *from-between* contributions equals 100%.

Fourth, to evaluate the relative importance of each variable/country/region/group in the global system/economy, we calculate the *net* contribution, which is defined as:

$$net = to\ between - from\ between.$$

Fifth, both the *to-between* and *net* contributions reflect the relative influence of variable/country/region/group in the global economy, and to evaluate the relative influence we need benchmarks, so I normalize *to-between* and *net* contributions by setting the US's contribution equal 1 in country/region cases and the oil's contribution equal 1 in the variable-group case, as demonstrated in Figures 7, 9 and 11.

Sixth, I separate the contribution of *oil price* from the contribution of the US (although it is estimated in the US model) because it is a truly global variable and its contribution should be analyzed in isolation.

Note: For now, I have only provided figures for 4 dimensions, i.e. aggregate, country, region, variable-group. We need to think how best to present the 176-variable results - tables or figures. I provide a very brief summary of important stylized observation/findings.

5.2.2 Aggregate Connectedness

Figure 5 plots the aggregate *from* contributions in different cases. Notice that in 176-variable case, there is no *from-within* contribution, we only have *own-* and *between*-variable contributions.

[Insert Figure 5 about here]

Own contribution is the same in the four sub-figures by construction, but the different *within* and *between* contributions in the country/region/variable-group cases present an interesting finding.

First, the *within* and *between* contributions exhibit similar patterns in the country and region cases. The *within* contribution is slightly bigger in the region case than in the country case by construction because we group several countries into regions. The *between* contribution in the country and region cases accounts for about 40% in the medium- to long-term and is relatively bigger than the *within* contribution.

Second, the patterns of *within* and *between* contributions in the variable-group case are quite different from those of the country and regions cases. In particular, the *within* contribution is much bigger than the *between* contributions, i.e. much of the spillover is among variables within the same group. So the large *between* contribution in the country and region cases is mainly attributable to the "between-country/region but within-variable-group" spillover, say between-country spillover from inflation to inflation, stock market to stock market, output to output. We need to elevate this point further?

5.2.3 Country Connectedness

Figures 6 and 7 plot the *from*, *to-between* and *net* contributions of 26 countries and oil price. There are some stylized findings.

[Insert Figures 6 and 7 about here]

First, in Figure 6, the larger the *between* contribution the larger the foreign influence on domestic economy. This figure shows that the EU, EU, China have the smallest *between* contribution, reflecting the fact that these are large and relatively close economies. (Brazil, Argentina, South Africa and Turkey also have relatively small *between* contribution - not sure if this is due to the nature of their economies or the set-up of our model).

Second, Japan, Switzerland and Saudi Arabia have the largest *between* contribution from the medium- to long-term, and this is likely due to the fact that these economies rely considerably on trade?Oil price clearly has considerable influence on Saudi Arabia.

Third, countries that are small and/or rely on considerable trade (export) tend to have bigger *between* contributions, e.g. Asian economies, Australia, Sweden, Norway.

Fourth, in Figure 7 if the red line is above (below) zero the country is net contributor (receiver). This figure shows that oil price, the US, EU are the largest net contributors in the system - and the US is clearly the most dominant economy. Argentina, Brazil and China are also net contributors. The net contribution of China is not big and this may reflect the fact that China mainly links with foreign economies through trade while its financial linkages with foreign economies are still limited (although we do not have Chinese stock market in our model) hence the international influence of China has yet to correspond to the size of its economies. The positive net contributions of Argentina and Brazil are attributed to the considerable *to-between* contributions of their inflations and interest rates to other Latin American economies (and Japan for the case of Brazil - is it because of the fact that Brazil is a big trading partner of Japan?). Meanwhile, Japan is net contributor in the very short-term but net receiver from the medium-term on. The rest are net receivers.

5.2.4 Region Connectedness

Figures 8 and 9 plot the *from*, *to-between* and *net* contributions of 12 regions and oil price.

[Insert Figures 8 and 9 about here]

We have similar observations/findings as in the country case, with the only difference now is the discussion in terms of regions. In particular, the US, EU, China and oil price are now only net contributors (Japan is a net contributor in the short-term). ASEAN economies are biggest net receivers (export-led growth countries).

5.2.5 Variable-Group Connectedness

Figures 10 and 11 plot the *from*, *to-between* and *net* contributions of 8 variable-groups. There are some stylized findings.

[Insert Figures 10 and 11 about here]

First, Figure 10 shows that oil price, exchange rate and stock market have the biggest *between* contribution (cross variable-group contribution). The *between* contributions in all other groups are relative small.

Second, Figure 10 shows that the *within* contribution (within variable-group contribution) is the biggest contribution as compared to *own* and *between* contribution. So most of the spillover happens within variable group.

Third, Figure 11 shows that oil price and stock market are the only net contributors.

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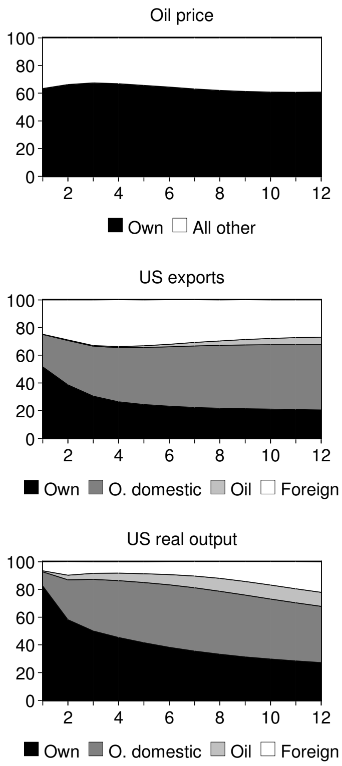

Figure 1: GFEVDs of US variables

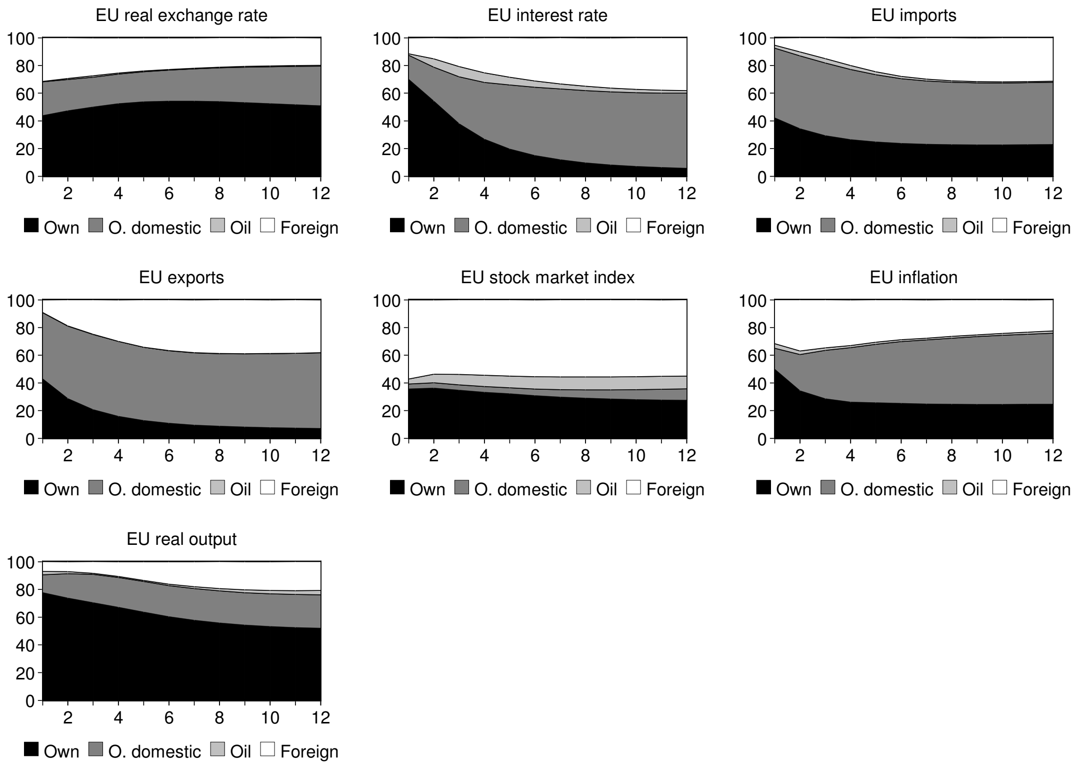

Figure 2: GFEVDs of EU variables

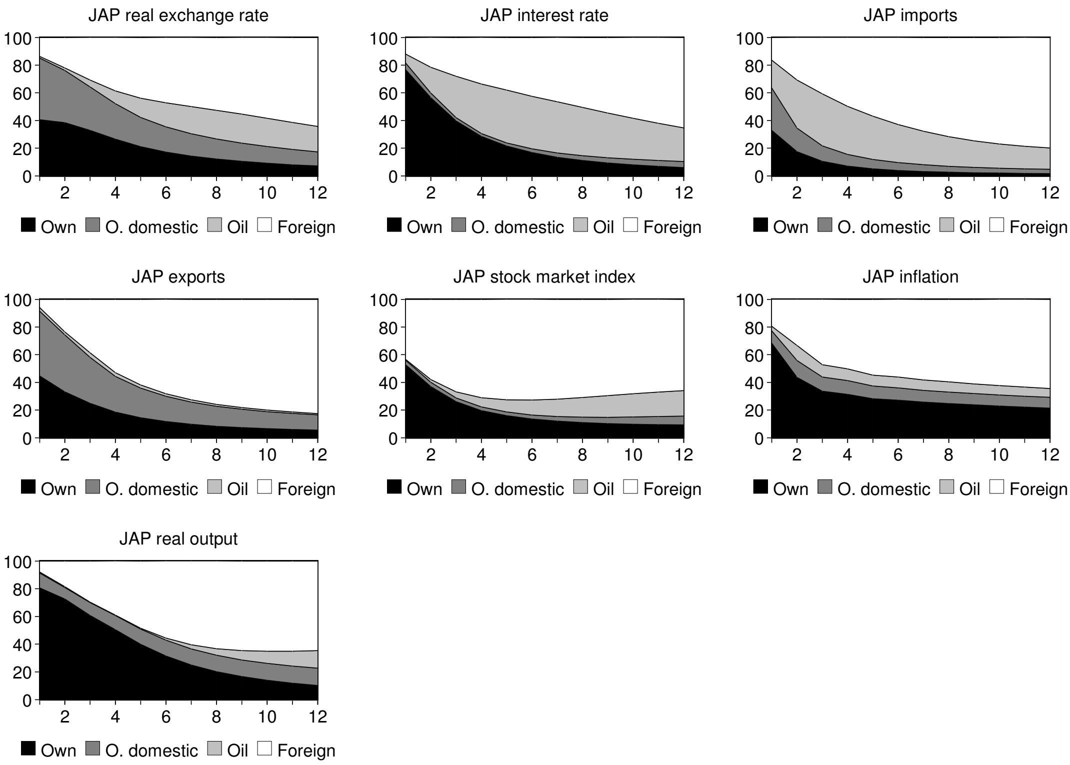

Figure 3: GFEVDs of Japan variables

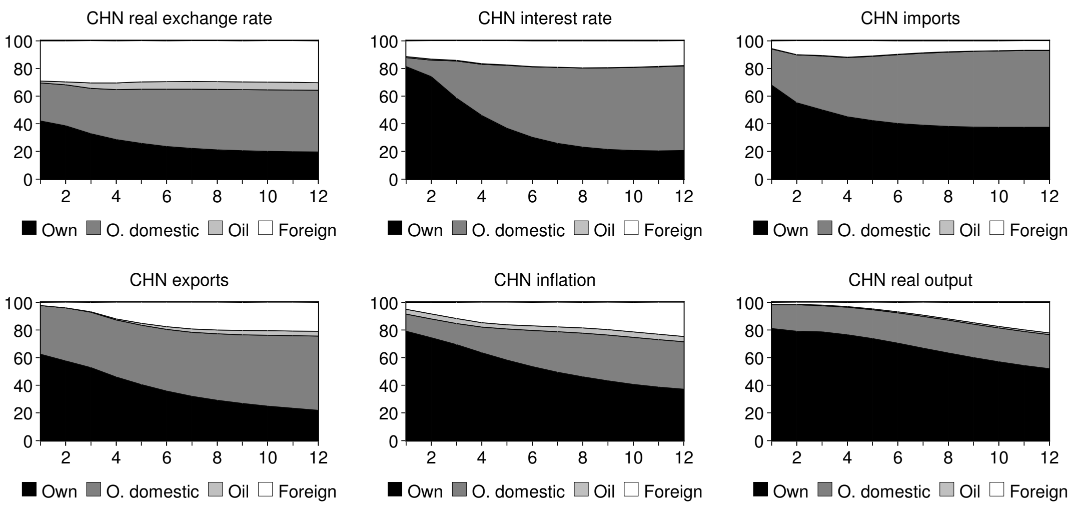

Figure 4: GFEVDs of China variables

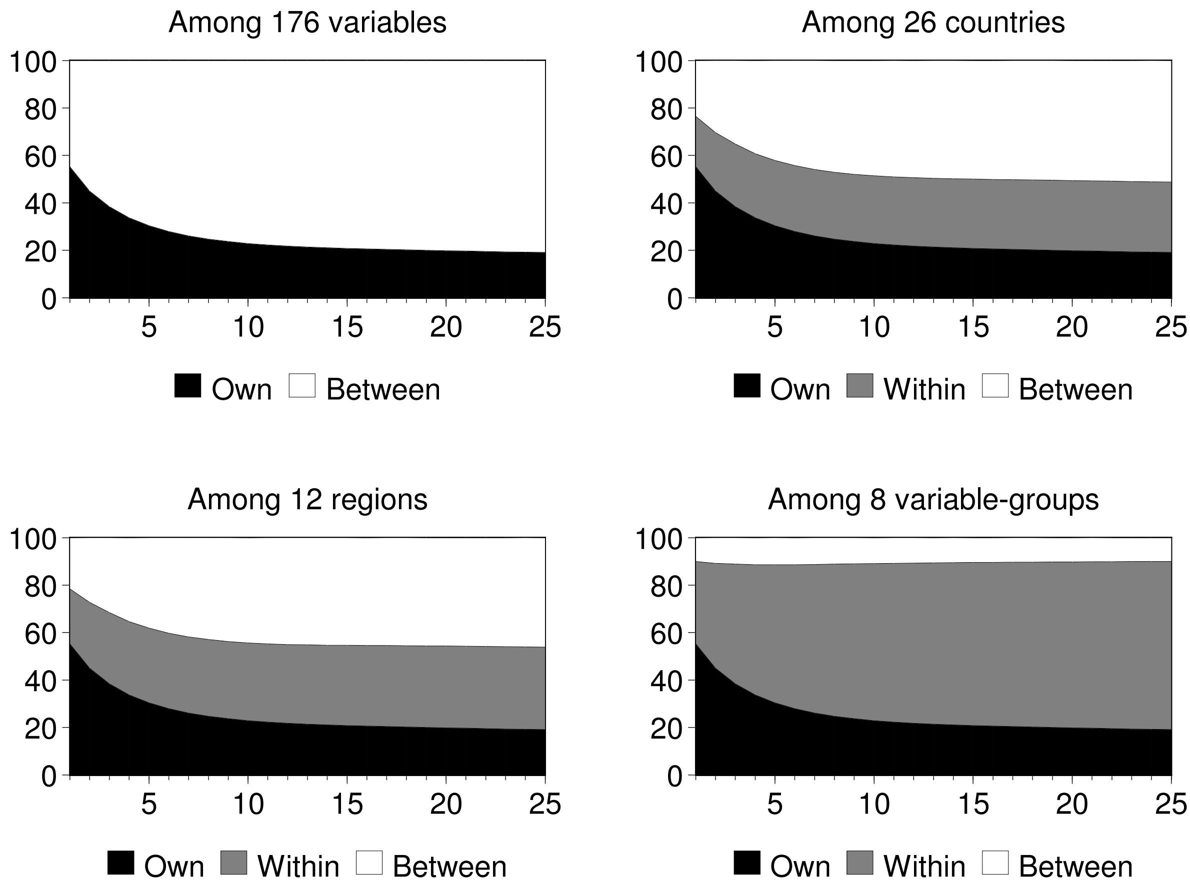

Figure 5: Aggregate Connectedness Measures

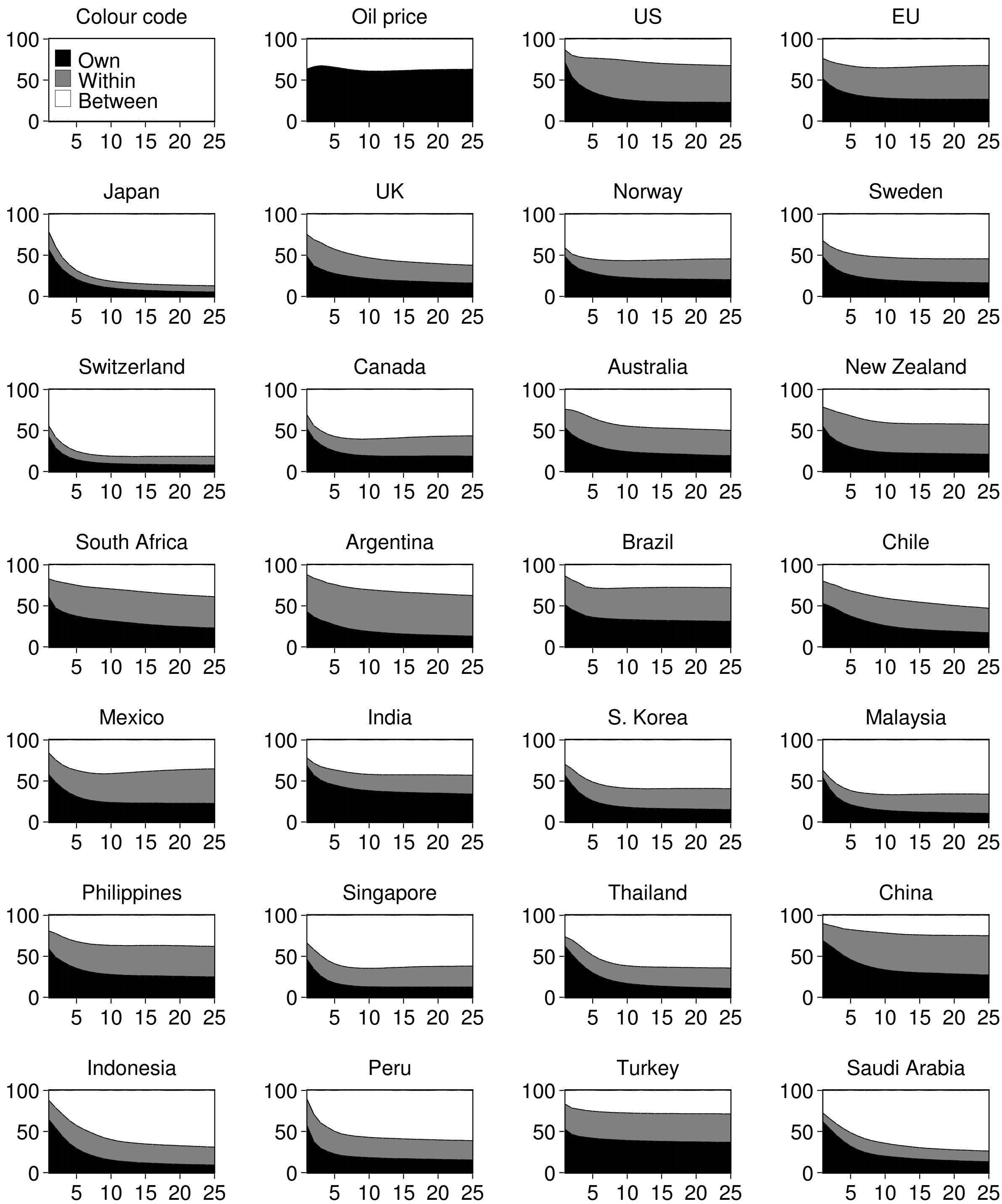

Figure 6: FROM Contributions among 26 Countries

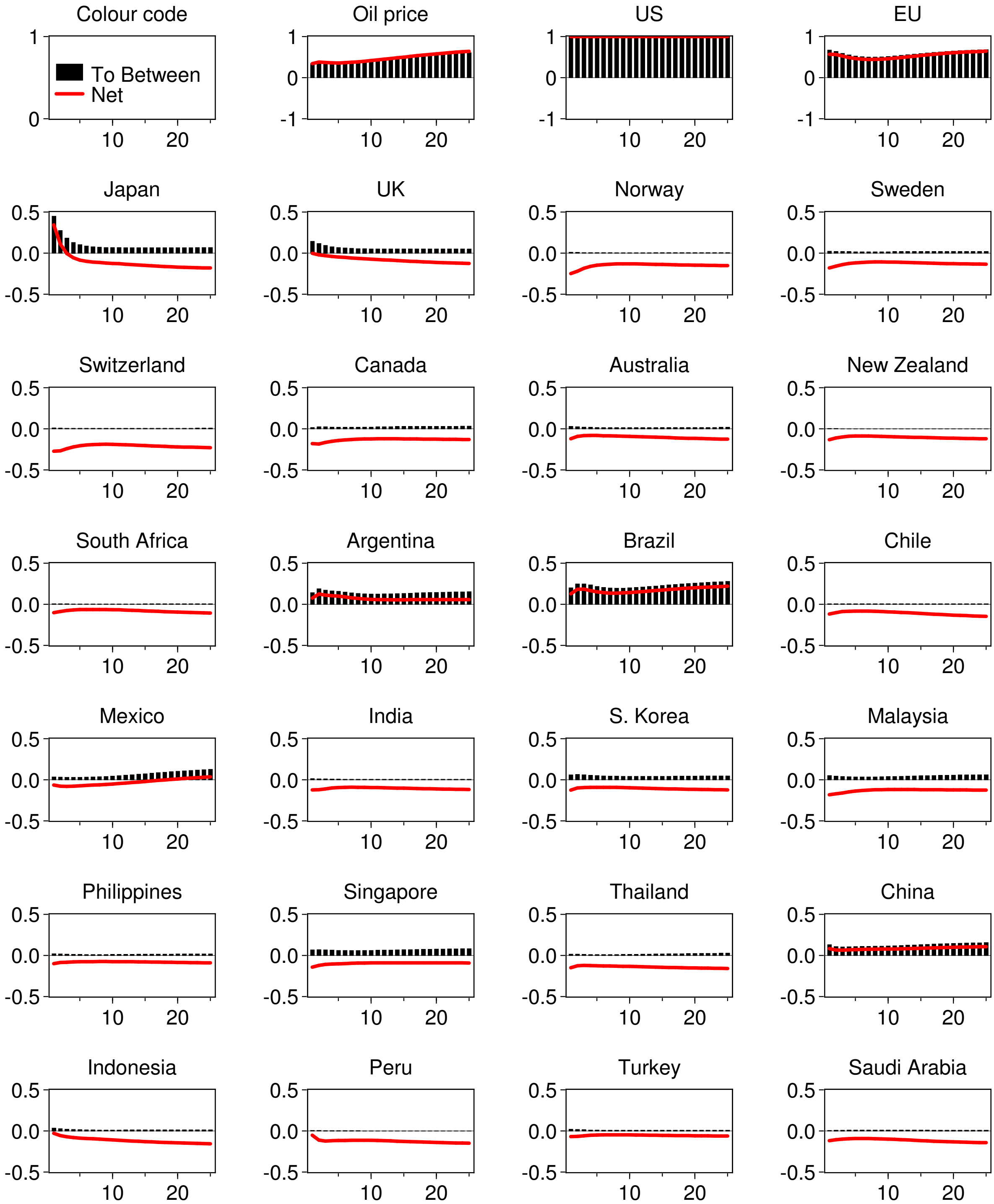

Figure 7: TO-BETWEEN and NET Contributions among 26 Countries (US = 1)

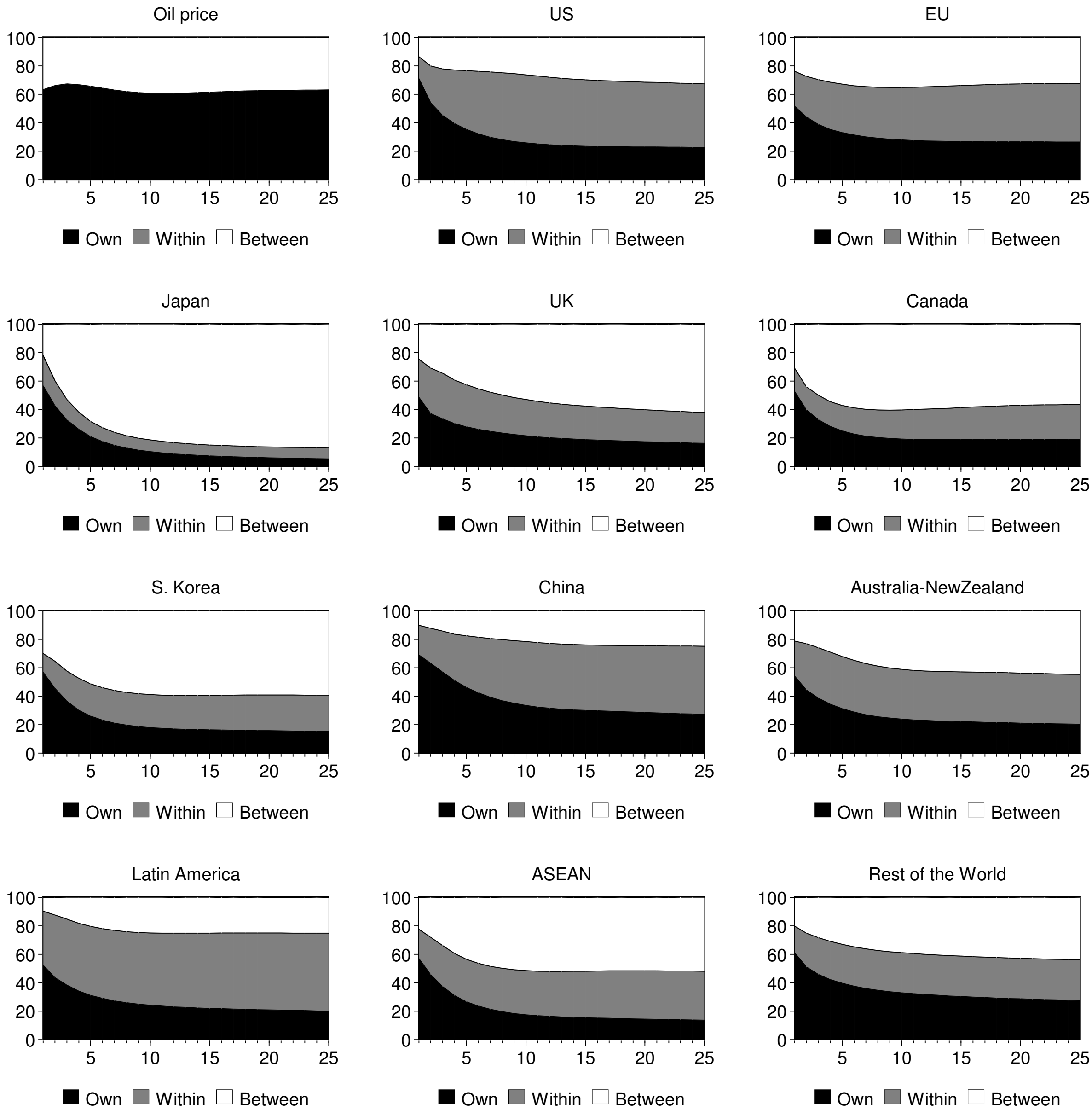

Figure 8: FROM Contributions among 12 Regions

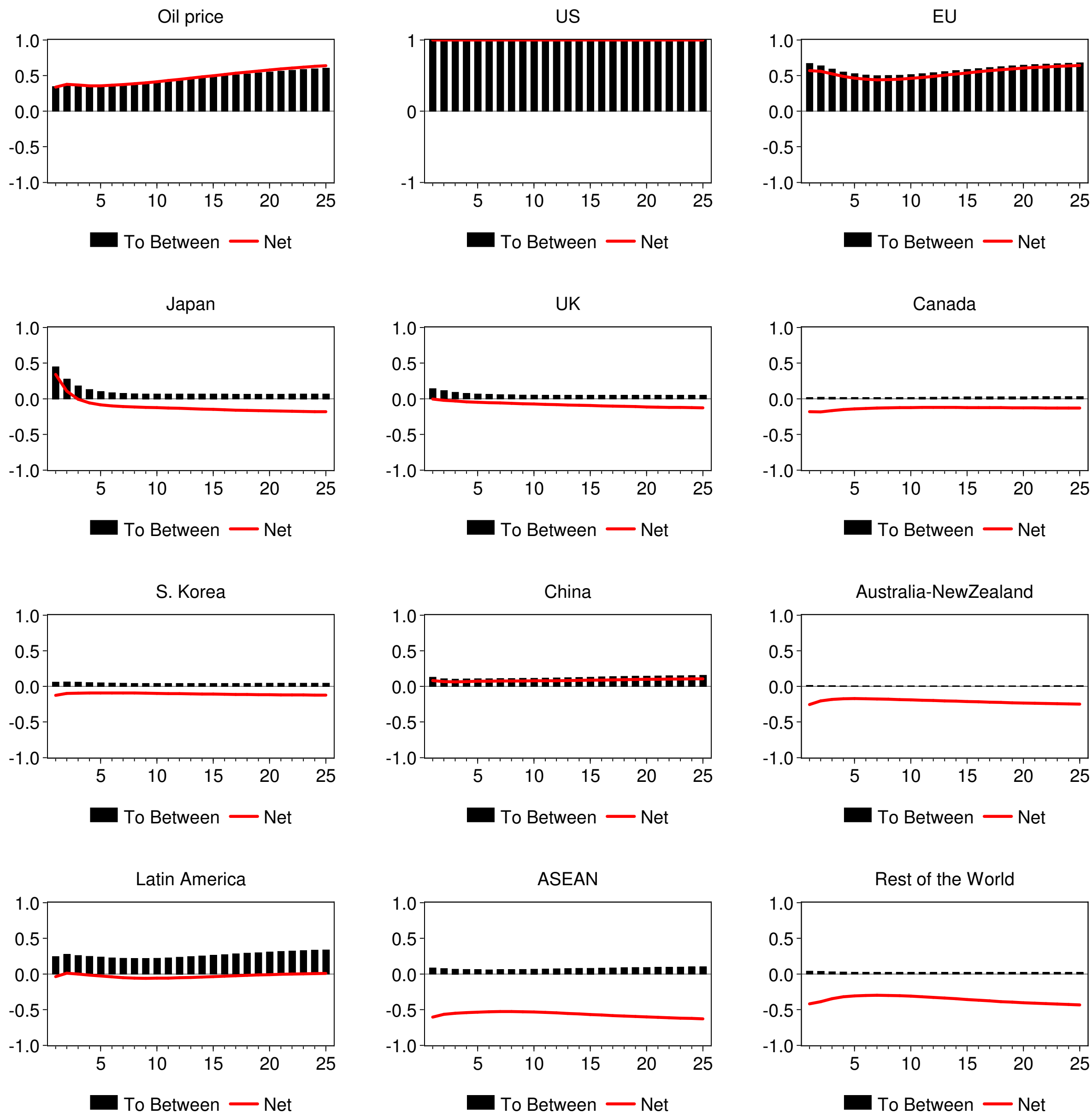

Figure 9: TO-BETWEEN and NET Contributions among 12 Regions (US = 1)

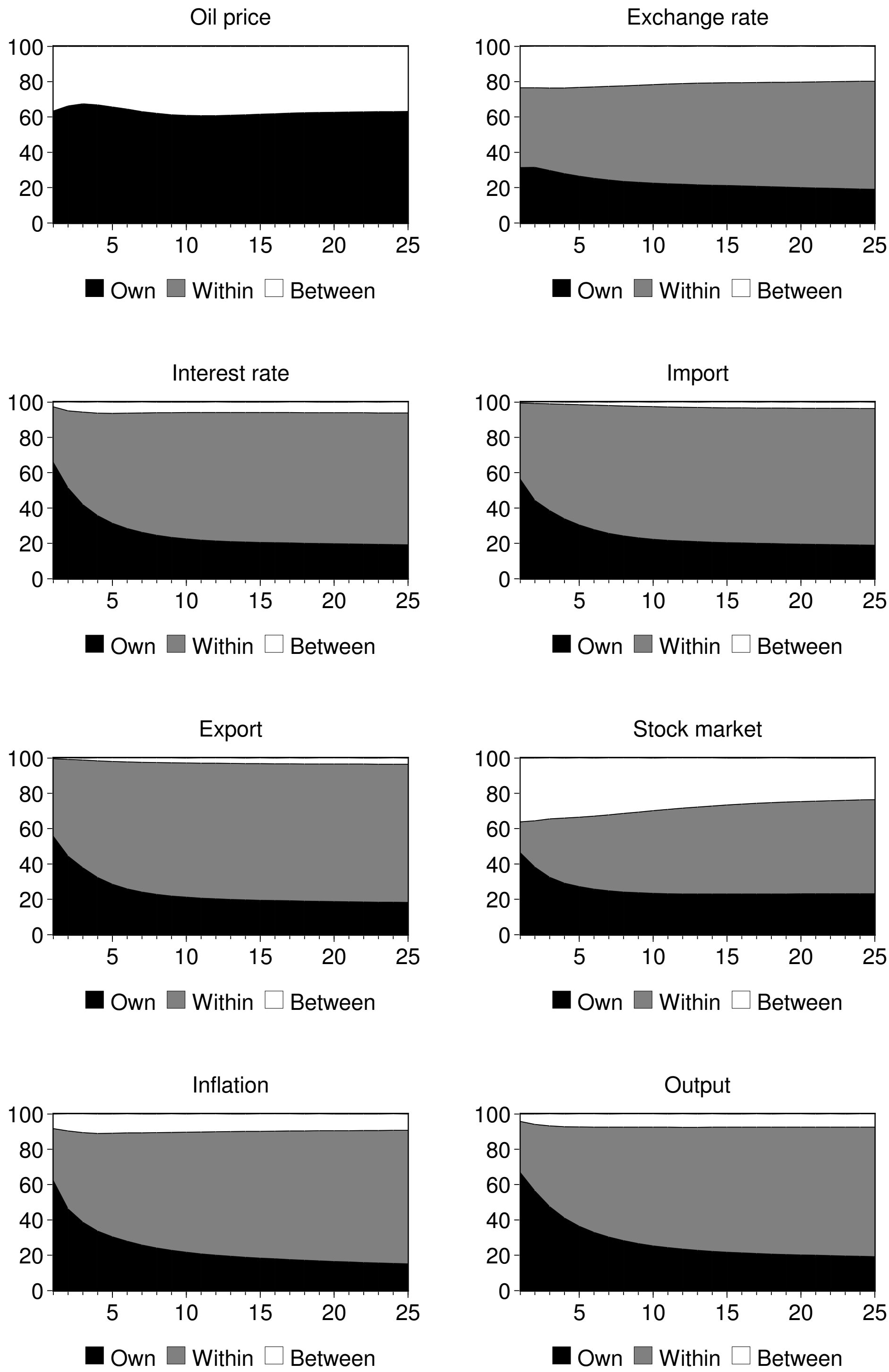

Figure 10: FROM Contributions among 8 Variable-Groups

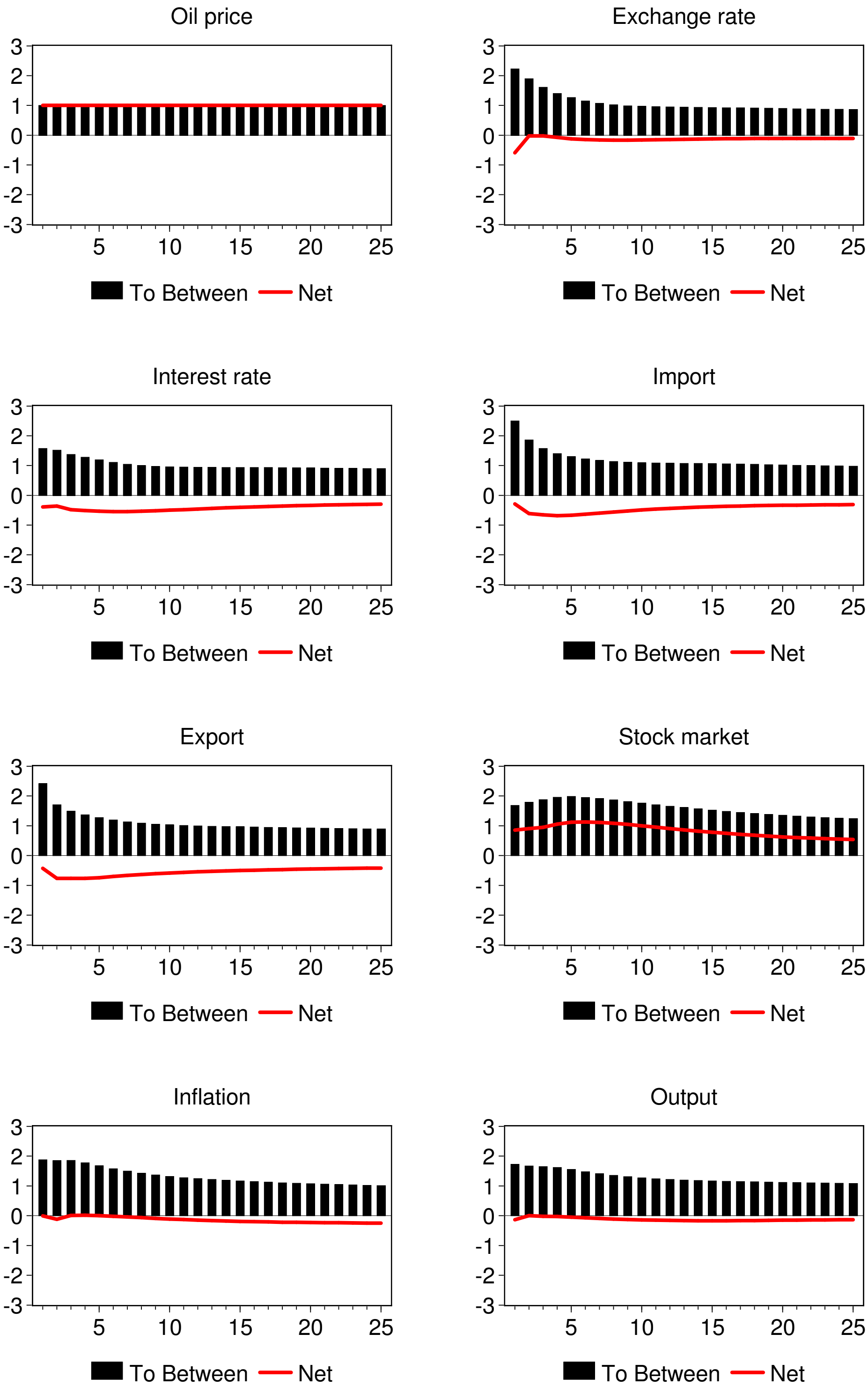

Figure 11: TO-BETWEEN and NET Contributions among 8 Variable-Groups (Oil = 1)