Structural Unemployment and Sectoral Migration After Trade Liberalization*

Shoya Ishimaru
University of Tokyo
e-mail: si81289@gmail.com

Soohyun (Catherine) Oh
Korea Institute for International Economic Policy
e-mail: shoh@kiep.go.kr

Seung-Gyu (Andrew) Sim
University of Tokyo
e-mail: sgsim@e.u-tokyo.ac.jp

First Draft: September, 2012

Abstract

This paper develops a dynamic general equilibrium model of international trade by incorporating search and matching friction and worker's sectoral migration decision into the traditional Heckscher-Ohlin framework. In particular, we shed lights on how long and how painful the transition is. Our simulation result shows that trade liberalization has positive effects on long-run output and employment, which is consistent with the prediction by the traditional model. However, it also reports that the trade reform may lower the output level, and increase unemployment rate and inequality in transition. The magnitude of the short-run welfare cost depends on the relative size of unemployed pool as long as the main source of it is the timing discrepancy between firms' employment decision and workers' migration decision. In particular, as the country has a relative larger size of unemployment pool, it incurs less short-run welfare cost because the large unemployment pool effectively absorbs the labor market congestion after trade reform.

Keywords: trade liberalization, search friction, transition dynamics
JEL Classification: F10, F11, F16, J64

*We have benefitted from discussions with Gihoon Hong and Jeongmeen Suh. Seung-Gyu Sim is very grateful to hospitality of Ross School of Business, University of Michigan. This research has been done during his visit at the University of Michigan. E. Han Kim and Seungjoon Oh deserve special recognition. Seung-Gyu Sim also thanks to the financial support of University of Tokyo (Young Researchers Overseas Program). The view point of this paper is those of the authors and does not necessarily reflect the view points of Korea Institute for International Economic Policy. All errors are our own.

1 Introduction

The traditional trade theory emphasizing comparative advantage stresses that trade liberalization makes all participating countries better-off in the long-run. However, in spite of the expected long-run 'gains from trade', trade liberalization is not always welcomed due to the public fear of a long and painful transition. Domestic firms may worry about whether they are driven out of business and workers worry about whether they are laid off after trade liberalization.[1] Until now, very little has been uncovered on what happens in transition and how long it takes to achieve the long-run gains. This paper develops a dynamic general equilibrium model by incorporating search and matching friction, proposed by Diamond (1982), Mortensen (1982), and Pissarides (1985), into the traditional Heckscher-Ohlin framework to examine those issues.

Although the trade theory based on comparative advantage points out that the (potentially painful) resource reallocation is the primary source of 'gains from trade', almost all models of international trade have assumed 'immediate full employment' or ignored the time to complete the reallocation process so far. Unlike the previous approach, this paper starts from the basic observation that it takes a non-negligible amount of time for an unemployed worker to find a job, and a worker does not quickly nor frequently change her/his occupation. When the search and matching friction meets with a multiple-sector trade model emphasizing comparative advantage, it causes structural unemployment as well as the usual frictional unemployment. Trade liberalization enforces workers to switch to the sectors with comparative advantage, but the sectoral migration decision by workers is not so flexible in reality because of cultural issue, specific human capital, and/or preferences.[2] We analyze the dynamic path in transition after trade reform by highlighting the worker flows.

Recently, there is a growing literature combining trade and search generated unemployment together. Helpman and Itskhoki (2010) develop a two-country two-sector model with search unemployment and firm heterogeneity. They show how variation in labor market frictions across countries affects trade patterns and unemployment. Felbermayr, Prat, and Schmerer (2011), using a monopolistic competition model, demonstrate that the labor force reallocation into relatively more productive firms improves the aggregate productivity, raises wage payment, and lowers the unemployment rate. In contrast, Helpman, Itskhoki, and Redding (2010), by introducing firms' selection, demonstrate that even though trade liberalization improves social welfare, the distribution of wages becomes more unequal and the level of unemployment can be higher in the trade equilibrium than in autarky. All those papers are based on the monopolistic competition framework proposed by Melitz (2003), where trade occurs even without comparative advantage. Also, they investigate only a static or steady state link between the tariff and unemployment rates without any short-run analysis.

Davidson, Martin, and Matusz (1999) introduce labor market friction in two-sector models of international trade in order to study the relationship between trade and unemployment. They treat labor market friction as the primary determinant of comparative advantage. They show that the country with better matching efficiency has

[1] Krugman (1993) also argues that one thing that both friends and foes of free trade seem to agree on is that the central issue is employment. Recently, Dutt, Mitra, and Ranjan (2009), using cross-nation panel data, find that trade liberalization increases unemployment rate in the short-run but lowers it in the long-run steady state.

[2] See Hayashi and Prescott (2008), Larch and Lechthaler (2011), and Kennan and Walker (2011).

comparative advantage on the sector with higher separation rate and consequently gets higher unemployment rate in a trade equilibrium because of the fast turnover rate in the specialized sector. Our approach is different from theirs. We take production technologies and factor endowments as the main determinants of comparative advantage. We extend the two-country two-sector Heckscher-Ohlin framework into a dynamic labor search model. Our starting point at the modeling stage is close to Bernard, Redding, and Schott (2007) in the sense that they also combine a labor search model and an international trade model with its emphasis on comparative advantage. Although they extend the search and trade framework by adding heterogeneity of countries, industries, and firms, they also analyze the steady state outcomes.

In our dynamic general equilibrium model, households provide the input factors, capital and labor, and consume two types of final goods, the labor intensive products and capital intensive products. Firms in each sector, the capital intensive sector and labor intensive sector, purchase capital and labor through each factor market and sell their final products in the final good market. The market participants are price-takers in all markets except the labor market. The labor market is subject to search and matching friction. Given their expectation on future (input and output) prices, firms decide how many units of capital they employs, and how many vacancies they create at every instant. Their employment decision determines the aggregate supply of outputs as well as the aggregate demand for inputs. The firms with vacancies search for unemployed workers and unemployed workers also look for jobs. Once they meet, they immediately start producing and share the surplus according to the Stole and Zwiebel (1996) bargaining rule. The dynamic worker flow through the matching and separation process determines the aggregate income distribution, which in turns determines the aggregate demand. It is required that the prices of capital and the final goods should clear the excess demand in each market in both countries at every instant.

In this paper, the tariff is recognized as a means of "import substitution industrialization strategy" as stated in Edwards (1993). The same type of goods are produced and traded in both countries and the prices are obtained by the market clearing conditions. The mutual tariff cut is expected to improve social efficiency in production. However, it mitigates the domestic protection for the comparative disadvantage sector, which causes structural unemployment and sectoral migration. In contrast, the monopolistic competition models assume that each firm produces its own differentiated product. Given 'love of variety' in consumer preferences, the role of the tariff as a protection device, is not properly captured, while the negative perception on the tariff as the source of cost is over-amplified. Furthermore, those models do not pay attention to the structural unemployment after trade reform and the induced sectoral migration.

Our approach presents some interesting implications. First, the dynamic worker flow affects the income distribution, aggregate demand, and the market clearing prices. In particular, if trade liberalization increases vacancies, employment, the aggregate income, and so the aggregate demand for each good, the implied *real inflation* due to the employment effect (or the aggregate income effect interchangeably) reduces the welfare gains from trade and increases consumption inequality across sectors across employment status. The employed worker in the exporting sector consumes more in spite of a higher price because their income goes up further, while other groups such as the unemployed workers consume less than before. The existing literature has effectively abstracted from applying the market-clearing price to the model by focusing on a partial equilibrium or assuming the perfectly elastic supply in one of the final goods (numeraire). Compared to our paper, those papers have risks of overestimating the gains from trade without the aggregate income effect and underestimating the consumption inequality.

Second, our model assumes that the unemployed workers possibly change their sectors when they are hit by revision shock.[3] Their switching decision depends on the value differentials between sectors as in Kennan and Walker (2011)[4]. When the tariff is removed or reduced, firms immediately respond to the environment changes and create more vacancies, while the unemployed workers in comparative disadvantage sector slowly switch to the sector with comparative advantage. The rigidity in the labor mobility creates fluctuations in the dynamic behavior of unemployment and welfare. Given discrepancy between the worker flow and vacancy creation, it is useful to think of introducing a grace period for a smooth transition. This paper demonstrates how it mitigates the short-run welfare cost to introduce a grace period.

Third, we present a generalized wage formula based on the Stole and Zwiebel (1996) bargaining rule. Helpman and Itskhoki (2010), Helpman, Itskhoki, and Redding (2010), and Felbermayr, Prat, and Schmerer (2011) also adopt the Stole and Zwiebel (1996) bargaining rule, but in a static or steady state environment. But we show that their argument is specific to the static or steady state environment and we alternatively provide a generalized wage formula. In particular, Felbermayr, Prat, and Schmerer (2011) argue that wages determined by the Stole and Zwiebel (1996) bargaining rule are proportional to the marginal product of labor and there is only one single wage even in the presence of continuum of heterogeneous firms. Hence in their model, all employed and unemployed workers receive the same level of wages and unemployment benefits, respectively. In contrast, the generalized formula rises with the marginal product of labor not in a linear fashion but in a convex fashion, which results in the sector- and time-specific wages, causes sectoral income inequality, fuels sectoral migration.

Our preliminary simulation results show that the mutual tariff cut increases the total production in each country and the world economy in the long-run. It lowers the steady-state unemployment rates in both countries. In light of these points, the prediction of our model is consistent with that of the traditional Heckscher-Ohlin model. In addition, we shed light on the employment effect (or the aggregate income effect interchangeably) which increases income, the demands for consumption goods, and the prices of the inputs and outputs in the long-run. The trade liberalization in our model is welfare-improving but also increases consumption inequality in the long-run welfare. At the day of announcement, the trade liberalization makes firms in the sector with comparative advantage optimistic. They create more vacancies, employ more inputs, and produce more from time zero. The firms in the other sector may need to downsize. But workers' sectoral migration does not respond quickly, which creates labor market congestion along the transition path. The discrepancy between the dynamic behaviors of vacancies and unemployment generates non-monotone transition paths, which requires substantial welfare cost and amplifies inequality in transition. In other words, because the discrepancy is the primary source of the short-run welfare cost, the country with a large initial unemployment pool can effectively absorb the

[3] It can be understood as occupation switching, sectoral migration, or geographic migration.

[4] They develop a tractable model of optimal migration, focusing on expected income as the main economic influence on migration.

impact of trade liberalization and incur less welfare cost.

The paper proceeds as follows. Section 2 presents the layout of the model. Section 3 and 4 report the simulation result of the long-run equilibrium and transition path, respectively. Based on the numerical experiments in section 3 and 4, we discuss the impact of a trade reform on welfare and policy implication of it in section 5. Then, we conclude in section 6.

2 The Model

2.1 Primitives

Consider a world economy consisting of two countries, home and foreign, with all foreign parameters and variables designated by a tilde ($\tilde{}$) on top of them. Each country is populated by continuum of two types of households, workers and investors. Workers provide labor and receive wages in the labor market. Investors provide capital and earn capital income in the capital market. In each country, there are two sectors, labor intensive sector ($i = 1$) and capital intensive sector ($i = 2$). Firms in each sector produce the sectoral products by purchasing capital and labor from the local factor markets. They can sell their products either in home or foreign markets. The markets for the final goods and the markets for capital are competitive in the sense that all market participants are price-takers. The labor markets in both countries are subject to search and matching friction as in Mortensen and Pissarides (1994). Time is continuous and both firms and households discount future at rate r. In what follows, given symmetry assumption, we proceed mainly with the home country when it is innocuous to do so.

Households There are L-measure of workers and ε-measure of investors in the home country and $\tilde{L}$-measure of workers, and $\tilde{\varepsilon}$-measure of investors in the foreign country. Each worker is endowed with one unit of labor in both countries, while the individual investor in home and foreign country is endowed with K/ε and $\tilde{K}/\tilde{\varepsilon}$ units of capital respectively. To embed comparative advantage based on endowment, let

$$L = 1, \quad K = 2, \quad \tilde{L} = 2, \quad \text{and} \quad \tilde{K} = 1.$$

It implies that home country is capital-abundant and the foreign country is labor-abundant. By construction, the home (foreign) country has comparative advantage on the capital (labor) intensive sector. All households retire or die at rate ρ and are replaced by the same types of newly-born households so that all measures remain unchanged.

The household with income flow w at time t consumes both goods (c_{1t}, c_{2t}) to maximize

$$(c_{1t}^{\frac{\sigma-1}{\sigma}} + c_{2t}^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

subject to the budget constraint $p_{1t}^c c_{1t} + p_{2t}^c c_{2t} = w$, where (p_{1t}^c, p_{2t}^c) represent the prices of the final goods paid by consumers at time t, and σ represents the elasticity of substitution. Note that $\sigma > 0$.[5] It is assumed that neither saving nor borrowing is allowed for any households. Solving the static utility maximization problem yields

$$c_{it} = \frac{(p^c_{it})^{-\sigma} w}{(p^c_{1t})^{1-\sigma} + (p^c_{2t})^{1-\sigma}}, \quad \text{where} \quad i \in \{1, 2\}. \tag{2}$$

The implied indirect utility flow from consumption is obtained by

$$\nu(p^c_{1t}, p^c_{2t}, w) = P_t^{-1} w, \quad \text{where} \quad P_t = \left((p^c_{1t})^{1-\sigma} + (p^c_{2t})^{1-\sigma} \right)^{\frac{1}{1-\sigma}}. \tag{3}$$

A worker is either employed or unemployed. A newly-born worker is unemployed and decides in which sector she starts her career. Let V_{it} and E_{it} be the lifetime value of unemployment in sector $i \in \{1, 2\}$ at time $t \in [0, \infty)$ and that of employment, respectively. The probability that a newly-born worker at time $t \in [0, \infty)$ chooses sector i is given by

$$\omega_{it} = \frac{\exp(\xi(V_{it} - V_{i't}))}{1 + \exp(\xi(V_{it} - V_{i't}))}, \quad \text{where} \quad i \neq i'. \tag{4}$$

Note that when $V_{it} = V_{i't}$, the worker is indifferent so that she chooses both sectors with equal probability. As the value differential $V_{it} - V_{i't}$ rises, she is more likely to choose sector i. An unemployed worker receives unemployment benefit b per instant and looks for a job offer. She also gets a chance to switch to the other sector $i' (\neq i)$ at rate μ_i. Once she gets the revision chance, she actually switches with probability ω_{it}. [6] The Hamilton-Jacobi-Bellman (HJB hereafter) equation for the unemployed worker in sector i is given by

$$rV_{it} = \nu(p^c_{1t}, p^c_{2t}, b) - \rho V_{it} + f(\theta_{it})(E_{it} - V_{it}) + \mu_i \omega_{i't}(V_{i't} - V_{it}) + \dot{V}_{it}, \tag{5}$$

where $f(\theta_{it})$ represents the job finding rate in sector i at time t.[7] The left-hand side can be interpreted as the opportunity cost of holding asset, unemployment in sector i at time t. The terms on the right-hand side represent the benefit flow from holding the asset V_{it} which consists of the dividend flow from the asset, the potential loss from retirement, the potential gains from job finding, the gains from switching, and the gains from changes in valuation of the asset, in order.

When she is employed in sector i, she receives wage flow w_{it} per instant. The employed worker retires at rate ρ and is separated from the job at rate δ due to an exogenous shock. The HJB equation for the employed worker in sector i is given by

$$rE_{it} = \nu(p^c_{1t}, p^c_{2t}, w_{it}) - \rho E_{it} + \delta(V_{it} - E_{it}) + \dot{E}_{it}. \tag{6}$$

Again, the left-hand side represents the opportunity cost of holding asset E_{it}. The right hand side consists of the dividend flow from the asset, the potential loss from

[5] As $\sigma \to 0, 1$, and ∞, the utility function in (1) converges to a Leontief function, Cobb-Douglas function, and von-Neumann function, respectively. See Klump and Preissler (2000).

[6] We may assume that the worker does not attempt to switch from the higher lifetime value to the other one or the unemployed worker incurs another switching cost in their moving. We will consider those cases later.

[7] It will be discussed later.

retirement, the loss from job separation, and the gains from changes in valuation of the asset, in order.

An investor sells their capital goods to a firm in either sector $i = 1$ or $i = 2$, and receives the market price of the capital goods, γ_{it}, per period. The detailed derivation of γ_i will be presented later. To abstract from another discussion on altruistic behavior, we simply assume that the capital good held by a retiree is immediately transferred to a newly born investor at no cost. Denote by I_{it} the lifetime values of the investor in sector i.

$$rI_{it} = \nu(p_{1t}^c, p_{2t}^c, \gamma_{it}) - \rho I_{it} + \dot{I}_{it} \tag{7}$$

Firms There is M_i measure of firms in sector i, in each country. Regardless of their geographic location, the firms in the same sector produce homogeneous consumption goods using the identical production technology. The production technology by the firm in sector i is given by

$$y_{it} = \alpha_i k_{it}^{1-\beta_i} h_{it}^{\beta_i}, \tag{8}$$

where $\beta_i \in (0, 1)$. The parameters α_i captures productivity of sector i. The operating profit in each sector is summarized by

$$R_{it} = p_{it}\alpha_i k_{it}^{1-\beta_i} h_{it}^{\beta_i} - \gamma_{it}k_{it} - w_{it}h_{it}. \tag{9}$$

The employed workers leave their firms at rate $(\rho + \delta)$. In order to hire workers, firms should create vacancies at cost η (per vacancy) and wait for job searchers due to the search and matching friction in the labor markets. Let v_{it} be the number of vacancies that the firm in sector i creates per instant. Each vacancy is filled with a worker at rate $q(\theta_{it})$. The measure of employees at a particular firm in sector i evolves as follows.

$$\dot{h}_{it} = -(\delta + \rho)h_{it} + q(\theta_{it})v_{it}, \quad \text{for each} \quad i = 1, 2. \tag{10}$$

The firm having $\overline{h}$ workers in sector i at time t sets a future employment plan (v_{is}, k_{is}) for each $s \in [t, \infty)$ to maximize

$$\int_t^\infty e^{-r(s-t)}(R_{is} - \eta v_{is})ds \tag{11}$$

subject to

$$\begin{aligned} \dot{h}_{is} &= -(\delta + \rho)h_{is} + q(\theta_{is})v_{is} \\ h_{it} &= \overline{h} \end{aligned}$$

The first order condition with respect to k_i implies that

$$\gamma_{it} = (1 - \beta_i)p_{it}\alpha_i k_{it}^{-\beta_i} h_{it}^{\beta_i} - \frac{\partial w_{it}}{\partial k_{it}} h_{it} \tag{12}$$

At every instant, the firm makes the factor-purchasing decision first and then it negotiates with workers at the production stage. Thus, when it makes the factor-purchasing decision, it should also consider how its decision affects the wage bargaining outcome, which is captured in the second term. Equation (12) shows the (individual) firm's demand for capital goods. In the factor market, the individual firm is a price-taker. Denote by $J_{it}(\overline{h})$ the expected value of the firm in sector i at time t having $\overline{h}$ workers. Also, let $J_{it}^h = \frac{\partial J_{it}}{\partial h_{it}}$. It represents the marginal value of labor at time t. Solving Hamiltonian yields

$$\eta \quad = \quad q(\theta_{it}) J_{it}^h \tag{13}$$

for each $t \in [0, \infty)$. The detailed derivation of (13) is postponed in Appendix A. In (13), the left-hand side represents the (marginal) cost of creating a vacancy and the right-hand side is the expected gain from creating the vacancy. Given optimal schedule of $\{k_{is}, v_{is}\}_{s \in [t, \infty)}$, we obtain

$$\pi_{it} = p_{it} \alpha_i k_{it}^{1-\beta_i} h_{it}^{\beta_i} - \gamma_{it} k_{it} - w_{it} h_{it} - \eta v_{it}, \tag{14}$$

and

$$\dot{J}_{it}^h = (r + \rho + \delta) J_{it}^h - p_{it} \frac{\partial y_{it}}{\partial h_{it}} + w_{it} + \frac{\partial w_{it}}{\partial h_{it}} h_{it}. \tag{15}$$

Wage Determination Wages are determined by the internal bargaining mechanism proposed by Stole and Zwiebel (1996). Let $\phi_i \in (0, 1)$ be the share of the joint surplus given to the employed worker in sector i. The firm keeps $(1 - \phi_i)$ portion of the surplus from each match. For each $i \in \{a, m\}$,

$$(1 - \phi_i)(E_{it} - V_{it}) = \phi_i J_{it}^h \text{ and } (1 - \phi_i)(\dot{E}_{it} - \dot{V}_{it}) = \phi_i \dot{J}_{it}^h \tag{16}$$

Combining (5), (6), (13), (15), and (16) altogether results in

$$\begin{aligned} [(1 - \phi_i) P_t^{-1} + \phi_i] w_{it} + \phi_i \frac{\partial w_{it}}{\partial h_{it}} h_{it} &= \phi_i p_{it} \frac{\partial y_{it}}{\partial h_{it}} \\ &+ (1 - \phi_i) b P_t^{-1} + \eta \phi_i \theta_{it} + (1 - \phi_i) \mu_i \omega_{i't} (V_{i't} - V_{it}) \end{aligned} \tag{17}$$

Note that the differential equation (17) should be true for all $t \in [0, \infty)$. Hence, solving the differential equation (17) and applying the undetermined coefficient method yields

$$w_{it} = \frac{\phi_i p_{it} (\partial y_{it} / \partial h_{it})}{(1 - \phi_i) P_t^{-1} + \phi_i \beta_i} + \frac{b P_t^{-1} + \eta \phi_i \theta_{it} / (1 - \phi_i) + \mu_i \omega_{i't} (V_{i't} - V_{it})}{P_t^{-1} + \phi_i / (1 - \phi_i)}. \tag{18}$$

Note that the last term in equation (18) does not depend on any of the input factors. It is interesting to compare (18) with the wage formula in Helpman and Itskhoki (2010), Helpman, Itskhoki, and Redding (2010), and Felbermayr, Prat, and Schmerer (2011). Since those papers investigate the steady state (or static game) only, they argue that wages are proportional to the marginal product of labor without the last term in (18). Indeed, they drop the time subscripts in their own differential equation similar to (17) and plug the guessed wage formula which is proportional to the marginal product of labor into (17). Then, using the undetermined coefficient method, they verify their initial guess. That's true when the second line in (17) is constant as in steady states. However, if the transition path *i.e.* dynamic behavior of θ_{it}, is also considered, it should be modified as in equation (18). The wage formula in (18) is not proportional to the marginal product of labor any more.

Factor Markets There are two different factor markets in each country, capital market and labor market. International borrowing or lending and international migration are precluded throughout the paper as in Bajona and Kehoe (2010). In each country, there is only one unified market for capital. All firms should buy capital goods through the capital market at the same price, *i.e* $\gamma_{1t} = \gamma_{2t} = \gamma_t$ at every instant.[8] The total supply of capital goods is fixed, while the demand for capital is described by (12) and (18). Equating the demand and supply yields

$$\gamma_t = \frac{(1-\phi_1)(1-\beta_1)p_{1t}\alpha_1}{(1-\phi_1)+\phi_1\beta_1 P_t} \cdot \frac{h_{it}^{\beta_1}}{k_{1t}^{\beta_1}} = \frac{(1-\phi_2)(1-\beta_2)p_{2t}\alpha_2}{(1-\phi_2)+\phi_2\beta_2 P_t} \cdot \frac{h_{2t}^{\beta_2}}{k_{2t}^{\beta_2}}. \tag{19}$$

Given $(p_{it}, p_{i't}, h_{it}, h_{i't})$, equation (19) determines $(\gamma_t, k_{it}, k_{i't})$ together with the endowment constraint.

The labor market in each country is segmented by sector. Each sectoral sub-market is subject to search and matching friction. Let u_{it} be the measure of unemployed workers in sector i at time t. Define the labor market tightness parameter in sector i at time t as

$$\theta_{it} := \frac{M_i v_{it}}{u_{it}}. \tag{20}$$

Given the constant returns to scale matching technology $m(M_i v_{it}, u_{it})$, the job-filling rate by a recruiting firm and the job-finding rate by a job searcher in sector i at time t are given by

$$q(\theta_{it}) = m(1, \theta_{it}^{-1}) \quad \text{and} \quad f(\theta_{it}) = m(\theta_{it}, 1) = \theta_{it} q(\theta_{it}). \tag{21}$$

In the numerical experiments later, we use a common Cobb-Douglas matching functions for all labor markets:

$$m(M_i v_{it}, u_{it}) = \lambda (M_i v_i)^{1-\kappa} u_i^{\kappa}, \quad \text{where} \quad \lambda > 0 \quad \text{and} \quad \kappa \in (0,1). \tag{22}$$

Denote by H_{it} the population employed in sector i at time t, respectively. By construction, $H_{it} = M_i h_{it}$ and $H_{1t} + H_{2t} + u_{1t} + u_{2t} = L$ at any time. The population size of each group evolves as follows.

$$\dot{H}_{it} = -(\rho+\delta)H_{it} + f(\theta_{it})u_{it} \tag{23}$$
$$\dot{u}_{it} = -(f(\theta_{it}) + \mu_i(1-\omega_{i't}) + \rho)u_{it} + \delta H_{it} + \mu_{i'}\omega_{it}u_{i't} + \rho\omega_{it}L. \tag{24}$$

In case of foreign country, $\tilde{L} = 2$ and all others remain same.

Product Markets Let us use superscript f to indicate the prices received by foreign firms. For example, p_{it}^f represents the price received by the foreign firm in sector i in the home market at time t, while $\tilde{p}_{it}^f$ represents the price received by the same firm

[8]It is also equivalent to assume that there are multiple segmented capital markets but the investors move freely between the segmented capital markets.

in the foreign market. Without loss of generality, we assume that the home country exports the capital intensive products $i = 2$ to the foreign country and imports the labor intensive products $i = 1$ from the foreign country. Then,

$$\frac{p_{1t}^c}{1+\tau} = \frac{p_{1t}}{1+\tau} = p_{1t}^f = \tilde{p}_{1t}^f = \tilde{p}_{1t}^c. \tag{25}$$

The first equality says that the price paid by domestic consumers is exactly same to the price received by domestic producers in home country regardless of tariff. The second equality captures the import constraint such that the foreign producers should receive less than the home producers in the home market in the presence of the tariff. The third says that the foreign firm receives the same price per unit in both home and foreign market. Otherwise, it will sell all products only in one market. The last equality implies that the price paid by foreign consumers should same to the price received by the foreign firms in foreign market regardless of tariff. Note that by construction, we obtain

$$\tilde{p}_{1t}^f < p_{1t} < (1+\tilde{\tau})p_{1t}, \tag{26}$$

for any $t \in [0, \infty)$. It implies that no domestic firms in the labor intensive sector ($i = 1$) export their products. By the same reasoning as above, we get

$$p_{2t}^c = p_{2t} = \tilde{p}_{2t} = \frac{\tilde{p}_{2t}^f}{1+\tilde{\tau}} = \frac{\tilde{p}_{2t}^c}{1+\tilde{\tau}} \quad \text{and} \quad p_{2t} < \tilde{p}_{2t}^f < (1+\tau)\tilde{p}_{2t}^f. \tag{27}$$

For expositional convenience, denote by W_t the aggregate income flow at time t.[9]

$$W_t = \sum_{i=1,2} [w_{it} H_{it} + u_{it} b] + \gamma_t K \tag{28}$$

Denote by χ_{2t} the exporting decision by the domestic firms in sector $i = 2$, the proportion of the capital intensive goods exporting to the foreign market. Along the same line, $\tilde{\chi}_{1t}$ represents the proportion of the foreign labor intensive products imported to the home market. By equating the aggregate demand and supply of agricultural products in each market, we obtain the market clearing condition as follows.

$$M_1 y_{1t} + \tilde{\chi}_{1t} \tilde{M}_1 \tilde{y}_{1t} = \frac{p_{1t}^{-\sigma} W_t}{p_{1t}^{1-\sigma} + p_{2t}^{1-\sigma}}, \quad \text{and} \tag{29}$$

$$(1 - \tilde{\chi}_{1t}) \tilde{M}_1 \tilde{y}_{1t} = \frac{(p_{1t}/(1+\tau))^{-\sigma} \tilde{W}_t}{(p_{1t}/(1+\tau))^{1-\sigma} + ((1+\tilde{\tau})p_{2t})^{1-\sigma}}. \tag{30}$$

By summing up the equations, (29) and (30), we get the world market clearing condition for the labor intensive products, which is free from the firm's allocation decision $\tilde{\chi}_{1t}$.

$$M_1 y_{1t} + \tilde{M}_1 \tilde{y}_{1t} = \frac{p_{1t}^{-\sigma} W_t}{p_{1t}^{1-\sigma} + p_{2t}^{1-\sigma}} + \frac{(p_{1t}/(1+\tau))^{-\sigma} \tilde{W}_t}{(p_{1t}/(1+\tau))^{1-\sigma} + ((1+\tilde{\tau})p_{2t})^{1-\sigma}} \tag{31}$$

[9]Note that if we consider 'entrepreneurs' who take the firm's profit as dividend and purchase final goods using the dividend, we should add the firm's profit onto the aggregate income flow. However, in all our simulations with reasonable parameter values, the dividend is negligible (less than 0.002).

The same argument is applied to the markets for the capital intensive products. By equating demand and supply in each market, we obtain

$$(1-\chi_{2t})M_2 y_{2t} = \frac{p_{2t}^{-\sigma} W_t}{p_{1t}^{1-\sigma} + p_{2t}^{1-\sigma}}, \tag{32}$$

$$\chi_{2t} M_2 y_{2t} + \tilde{M}_2 \tilde{y}_{2t} = \frac{((1+\tilde{\tau})p_{2t})^{-\sigma}\tilde{W}_t}{(p_{1t}/(1+\tau))^{1-\sigma} + ((1+\tilde{\tau})p_{2t})^{1-\sigma}}, \quad \text{and} \tag{33}$$

$$M_2 y_{2t} + \tilde{M}_2 \tilde{y}_{2t} = \frac{p_{2t}^{-\sigma} W_t}{p_{1t}^{1-\sigma} + p_{2t}^{1-\sigma}} + \frac{((1+\tilde{\tau})p_{2t})^{-\sigma}\tilde{W}_t}{(p_{1t}/(1+\tau))^{1-\sigma} + ((1+\tilde{\tau})p_{2t})^{1-\sigma}}. \tag{34}$$

Combining (31) and (34) yields (p_{1t}, p_{2t}). Then, plugging (p_{1t}, p_{2t}) into (30) and (33) yields $(\chi_{2t}, \tilde{\chi}_{1t})$.

Trade Equilibrium We finally finish this section by defining the trade equilibrium of our interest. The following definition summarizes the overall shape of our model.

A *trade equilibrium* for the world economy with tariff $(\tau, \tilde{\tau})$ consists of bounded time series of choice rules $\{c_{it}, \tilde{c}_{it}, k_{it}, \tilde{k}_{it}, v_{it}, \tilde{v}_{it}\}_{i=1,2}$, labor market tightness parameters $\{\theta_{it}, \tilde{\theta}_{it}\}_{i=1,2}$, price vector $\{p_{it}, \tilde{p}_{it}, \gamma_{it}, \tilde{\gamma}_{it}, w_{it}, \tilde{w}_{it}\}$, profit flow $\{\pi_{it}, \tilde{\pi}_{it}\}_{i=1,2}$, value equations $\{E_{it}, \tilde{E}_{it}, V_{it}, \tilde{V}_{it}, J_{it}^h, \tilde{J}_{it}^h\}_{i=1,2}$, and measures $\{H_{it}, \tilde{H}_{it}, h_{it}, \tilde{h}_{it}, u_{it}, \tilde{u}_{it}\}_{i=1,2}$ at every $t \in [0, \infty)$ such that:

- (i) Each household in home (foreign) country optimally chooses $\{c_{1t}, c_{2t}\}$ $(\{\tilde{c}_{1t}, \tilde{c}_{2t}\})$ at every t.
- (ii) Each firm in sector i in home (foreign) country optimally chooses $\{k_{it}, v_{it}\}$ $(\{k_{it}, v_{it}\})$ at every t. It also determines $\{\pi_{it}, \tilde{\pi}_{it}\}_{i=1,2}$ at every t.
- (iii) The aggregate consistency requires that the vacancy creation decision by the individual firm based on (13) should be consistent with the definition of market tightness $\{\theta_{it}, \tilde{\theta}_{it}\}_{i=1,2}$ in (20) at every t.
- (iv) The world market clearing conditions in (31) and (34) and the wage setting rule in (18) jointly determine $\{p_{it}, \tilde{p}_{it}, \gamma_{it}, \tilde{\gamma}_{it}, w_{it}, \tilde{w}_{it}\}_{i=1,2}$ at every t. By construction, $p_{1t} = (1+\tau)\tilde{p}_{1t}$, $(1+\tilde{\tau})p_{2t} = \tilde{p}_{2t}$, $\gamma_{1t} = \gamma_{2t}$, and $\tilde{\gamma}_{1t} = \tilde{\gamma}_{2t}$ at every t.
- (v) The evolution of the entire system is recursively governed by the law of motion of (5), (6), (15), (23), and (24) given $\{E_{i0}, \tilde{E}_{i0}, V_{i0}, \tilde{V}_{i0}, J_{i0}^h, \tilde{J}_{i0}^h\}_{i=1,2}$ and $\{H_{i0}, \tilde{H}_{i0}, u_{i0}, \tilde{u}_{i0}\}_{i=1,2}$.[10]

3 Steady State Analysis

In this section, we characterize the steady state equilibrium and provide some illustrative simulation experiments.[11] In fact, the long-run steady state impact of trade liberalization (or the mutual tariff cut) is potentially ambiguous and sensitive to the choice of parameter values. Our calibration follows common practice in the literature.

[10] By the law of motion in (6), one can easily find out that $H_{it} = M_i h_{it}$ at every t.

[11] Hopenhayn (1992) proves the existence and uniqueness of the long-run stationary (steady state) equilibrium in one-country one-good, one-factor set up.

3.1 Steady State Characterization

All values on steady states are denoted without time subscript throughout the paper. It is required that the law of motion described in equilibrium condition (v) in the previous section should be stationary on steady states. It implies that the differential equations described in (5), (6), (15), (23), and (24) should be constant overtime:

$$E_{it} = E_i \quad V_{it} = V_i \quad J_{it}^h = J_i^h \quad H_{it} = H_i \quad u_{it} = u_i$$
$$\tilde{E}_{it} = \tilde{E}_i \quad \tilde{V}_{it} = \tilde{V}_i \quad \tilde{J}_{it}^h = \tilde{J}_i^h \quad \tilde{H}_{it} = \tilde{H}_i \quad \tilde{u}_{it} = \tilde{u}_i$$

Since $H_{it} = M_i h_{it}$, we get $h_{it} = h_i = H_i/M_i$. Suppose that the domestic prices for the final goods are given. Then, $(p_1, p_2) = ((1+\tau)\tilde{p}_1, \tilde{p}_2/(1+\tilde{\tau}))$. The individual firm having h_i in sector i optimally chooses k_i units of capital as described in (12), which results in

$$\gamma = \frac{\alpha_1(1-\beta_1)(1-\phi_1)p_1}{(1-\phi_1)+\phi_1\beta_1 P}\Big(\frac{h_1}{k_1}\Big)^{\beta_1} = \frac{\alpha_2(1-\beta_2)(1-\phi_2)p_2}{(1-\phi_2)+\phi_2\beta_2 P}\Big(\frac{h_2}{k_2}\Big)^{\beta_2}. \tag{35}$$

Given $\{p_i, h_i\}_{i=1,2}$, equation (35) and the feasibility constraint, $K = M_1k_1 + M_2k_2$, jointly determine (γ, k_1, k_2) and $(\tilde{\gamma}, \tilde{k}_1, \tilde{k}_2)$. The steady state values of $\{J_i^h, \tilde{J}_i^h\}_{i=1,2}$ are obtained by combining (13), (15), and (18) and applying the steady state conditions, which results in

$$\Big(\frac{k_i}{h_i}\Big)^{1-\beta_i} = \frac{(1-\phi_i)+\phi_i\beta_i P}{\alpha_i\beta_i(1-\phi_i)p_i}\Big[\frac{\eta(r+\rho+\delta)}{q(\theta_i)} + \frac{bP^{-1} + \frac{\eta\phi_i\theta_i}{(1-\phi_i)} + \mu_i\omega_{i'}(V_{i'}-V_i)}{P^{-1}+\phi_i/(1-\phi_i)}\Big] \tag{36}$$

Given $\{p_i, h_i, \tilde{h}_i, V_i, \tilde{V}_i, k_i, \tilde{k}_i\}_{i=1,2}$, equation (36) solves for $\{\theta_i, \tilde{\theta}_i\}_{i=1,2}$, which also returns $\{v_i, \tilde{v}_i\}$. In turn, given $\{p_i, h_i, \tilde{h}_i, V_i, \tilde{V}_i, k_i, \tilde{k}_i, \theta_i, \tilde{\theta}_i\}_{i=1,2}$, we obtain $\{w_i, \tilde{w}_i\}_{i=1,2}$ using

$$w_i = \frac{\phi_i p_i(\partial y_i/\partial h_i)}{(1-\phi_i)P^{-1}+\phi_i\beta_{ih}} + \frac{bP^{-1}+\eta\phi_i\theta_i/(1-\phi_i)+\mu_i\omega_{i'}(V_{i'}-V_i)}{P^{-1}+\phi_i/(1-\phi_i)}. \tag{37}$$

Then, the steady state value equations are described by

$$rJ_i^h = \beta_i p_i\alpha_i k_i^{1-\beta_i}h_i^{\beta_i-1} - w_i - w_i^h h_i - (\rho+\delta)J_i^h, \tag{38}$$
$$rE_i = \nu(p_1^c, p_2^c, w_i) - \rho E_i + \delta(V_i - E_i), \quad \text{and} \tag{39}$$
$$rV_i = \nu(p_1^c, p_2^c, b) - \rho V_i + f(\theta_i)(E_i - V_i) + \mu_i\omega_{i'}(V_{i'} - V_i), \tag{40}$$

which should be consistent with the initial values. Also, consistency requires that

$$0 = -(\rho+\delta)H_i + f(\theta_i)u_i, \quad \text{and} \tag{41}$$
$$0 = -(f(\theta_i)+\mu_i\omega_{i'}+\rho)u_i + \delta H_i + \mu_{i'}\omega_i u_{i'} + \rho\omega_i L \tag{42}$$

restores the invariant measures of $\{H_i, \tilde{H}_i, u_i, \tilde{u}_i\}_{i=1,2}$. Finally, the price vector of (p_1, p_2) is required to clear the world market as described in (31) and (34).

3.2 Simulation Results: Steady State

This subsection illustrates the effects of trade liberalization by simulating our model under different magnitude of tariff. The parameter values and the numerical algorithms adopted in the baseline model are postponed in Appendix B. [Table 1] shows how the long-run steady state equilibrium responds to the different levels of tariff.

Table 1: Steady State Outcomes

	Home		Foreign	
	labor intensive Sector	capital intensive Sector	labor intensive Sector	capital intensive Sector
	Panel A: $\tau = \tilde{\tau} = 0.05$			
y_i	0.171	1.288	0.791	0.602
$y_1 + y_2$	1.459		1.393	
χ_i	-	0.234	0.306	-
k_i	0.161	1.839	0.439	0.561
h_i	0.178	0.754	1.172	0.669
p_i	1.329	1.058	1.266	1.110
w_i	1.085	1.294	0.723	0.718
γ_i	0.200	0.200	0.320	0.320
v_i	0.007	0.072	0.059	0.033
u_i	0.017	0.051	0.101	0.058
θ_i	0.422	1.426	0.588	0.571
u/L	0.067		0.079	
E_i	53.082	61.910	34.235	34.044
V_i	52.798	61.228	33.875	33.691
EV	60.149		34.137	
	Panel B: $\tau = \tilde{\tau} = 0.04$			
y_i	0.154	1.306	0.803	0.593
$y_1 + y_2$	1.460		1.395	
χ_i	-	0.253	0.332	-
k_i	0.142	1.858	0.448	0.552
h_i	0.163	0.770	1.185	0.659
p_i	1.410	1.130	1.356	1.175
w_i	1.147	1.393	0.782	0.770
γ_i	0.208	0.208	0.328	0.328
v_i	0.006	0.075	0.063	0.033
u_i	0.016	0.051	0.100	0.057
θ_i	0.348	1.473	0.632	0.587
u/L	0.067		0.078	
E_i	52.774	62.497	34.789	34.303
V_i	52.527	61.799	34.409	33.943
EV	60.717		34.586	

Table 1: Steady State Outcomes (Contd.)

	Home		Foreign	
	labor intensive Sector	capital intensive Sector	labor intensive Sector	capital intensive Sector
		Panel C: $\tau = \tilde{\tau} = 0.03$		
y_i	0.136	1.326	0.812	0.586
$y_1 + y_2$	1.462		1.398	
χ_i	-	0.270	0.358	-
k_i	0.123	1.877	0.455	0.545
h_i	0.146	0.787	1.195	0.652
p_i	1.549	1.249	1.504	1.287
w_i	1.258	1.561	0.879	0.859
γ_i	0.218	0.218	0.340	0.340
v_i	0.004	0.080	0.068	0.034
u_i	0.016	0.052	0.098	0.055
θ_i	0.272	1.541	0.691	0.623
u/L	0.067		0.077	
E_i	52.648	63.414	35.463	34.723
V_i	52.441	62.693	35.058	34.347
EV	61.638		35.171	
		Panel D: $\tau = \tilde{\tau} = 0.02$		
y_i	0.116	1.349	0.816	0.583
$y_1 + y_2$	1.464		1.400	
χ_i	-	0.287	0.384	-
k_i	0.102	1.898	0.458	0.542
h_i	0.126	0.808	1.200	0.650
p_i	1.808	1.462	1.773	1.492
w_i	1.470	1.868	1.054	1.025
γ_i	0.233	0.233	0.359	0.359
v_i	0.003	0.086	0.074	0.037
u_i	0.015	0.052	0.096	0.054
θ_i	0.191	1.641	0.773	0.687
u/L	0.067		0.075	
E_i	52.809	64.859	36.341	35.416
V_i	52.649	64.105	35.902	35.013
EV	63.127		35.984	

3.2.1 Without Reverse Migration

In fact, the specification of ω_{it} in (4) allows the reverse flow from the sector with a larger value of unemployment to the other sector with a smaller value, that is, the reverse flow from the sector with comparative advantage to the sector with disadvantage. Since it does not have a significant meaning, we assume that when $V_{i'} < V_i$, $\mu_i = 0$. Instead, in the next subsection, we show the results with the reverse sectoral migration which are not so different from the results in this subsection at least in the qualitative sense.

Prices In a static environment with full employment, trade liberalization (the mutual tariff cut) is expected to lower the price paid by consumer and raise the price received by producers. In addition to this, in a dynamic environment considering the frictional labor market, the increased producer surplus gives firms incentives to create more vacancies, employ more inputs, and produce more. An increase in employment results in higher aggregate income in the economy. The aggregate income effect shifts out the demands for consumption goods. As long as factor reallocation driven by trade liberalization has a limited effect on total output, the aggregate supply is not so elastic.[12] Then, the employment effect (the aggregate income effect interchangeably) raises the long-run equilibrium prices of final goods. In turns, it also improves the marginal product of input factors and raises the prices of them. The *long-run real inflation* reduces the welfare gains from trade. In fact, [Table 1] reports that the output prices are $(p_1, p_2) = (1.81, 1.46), (1.55, 1.25), (1.41, 1.13), (1.33, 1.06)$ at each tariff level of $(0.02, 0.03, 0.04, 0.05)$. As the tariff declines, the long-run equilibrium wages, $(w_1, w_2, \tilde{w}_1, \tilde{w}_2)$, grow up from $(1.09, 1.29, 0.72, 0.72)$ to $(1.47, 1.87, 1.05, 1.03)$. At the same time, the prices of capital $(\gamma, \tilde{\gamma})$ rise from $(0.20, 0.32)$ to $(0.23, 0.36)$. [Table 2] summarizes this.

Interestingly, each input factor is always more expensive in the the country where it is scarce. It is also noteworthy that the steady-state price of final goods and wages show considerable increases than the price of capital in response to trade liberalization. The same patterns are observed in both countries.

Sectoral Migration and Output Share As the tariff is reduced in [Table 2], wages in the comparative advantage sector grow faster than the other sector in home, while those in the comparative advantage sector grow slower in the foreign country. As a result, when the tariffs are given by $(0.02, 0.03, 0.04, 0.05)$, $V_2 - V_1 = (11.46, 10.25, 9.27, 8.43)$, but $\tilde{V}_1 - \tilde{V}_2 = (0.89, 0.71, 0.47, 0.18)$ in [Table 1]. This implies that as the tariff is reduced, workers are more likely to switch to the sector with comparative advantage in both countries. In fact, both $h_2 + u_2$ in the home country and $\tilde{h}_1 + \tilde{u}_1$ in the foreign country decline with the tariff levels.

When the tariff rates are fixed at $(0.02, 0.03, 0.04, 0.05)$, the total outputs are given by $(1.464, 1.462, 1.460, 1.459)$ in home, $(1.400, 1.398, 1.395, 1.393)$ in the foreign country, and $(2.864, 2.860, 2.856, 2.852)$ in total, respectively. It tells us that trade liberalization increases the total production. The steady-state production becomes concentrated on the comparative advantage sector in the home country, but not in the

[12] Refer to the fourth lines in each panel. The changes in output y is not substantial.

Table 2: The Employment Effect (The Aggregate Income Effect)

τ	$h_1 + h_2$	w_1	w_2	γ	Aggregate Income
			Home		
0.02	0.933	1.470 (1.379)	1.868 (1.490)	0.233 (1.197)	1.713(1.440)
0.03	0.933	1.258 (1.178)	1.561 (1.233)	0.218 (1.106)	1.432 (1.203)
0.04	0.933	1.147 (1.067)	1.393 (1.090)	0.208 (1.044)	1.279(1.075)
0.05	0.933	1.085 (1)	1.294 (1)	0.200 (1)	1.190 (1)
			Foreign		
0.02	1.851	1.054 (1.458)	1.025 (1.426)	0.359 (1.120)	1.954 (1.445)
0.03	1.847	0.879 (1.216)	0.859 (1.196)	0.340 (1.060)	1.634 (1.208)
0.04	1.844	0.782 (1.082)	0.770 (1.072)	0.328 (1.023)	1.458 (1.078)
0.05	1.841	0.723 (1)	0.718 (1)	0.320 (1)	1.352 (1)

other country. The output shares of the comparative advantage products are given by $(0.921, 0.907, 0.894, 0.883)$ in the home country, and $(0.583, 0.581, 0.575, 0.568)$ in the foreign country at each tariff level of $(0.02, 0.03, 0.04, 0.05)$. The output share of the comparative advantage products consistently declines with tariff. The exporting shares in the sector with comparative advantage monotonically decline with the tariff rate from 28.7% to 23.4% in home and from 38.4% to 30.6% in the foreign country.

Labor Market The labor market can be summarized by three key variables; vacancy, unemployment, and v/u ratio. In particular, the steady state measure of vacancy and the v/u ratios in the exporting sector decline with the level of tariff. When the tariff rates are given by $(0.02, 0.03, 0.04, 0.05)$, the steady state measure of vacancy in the exporting sector are $(0.086, 0.080, 0.075, 0.072)$ in home and $(0.074, 0.068, 0.063, 0.059)$ in the foreign country. Also, we observe that the v/u-ratios in the exporting sector are $(1.64, 1.54, 1.47, 1.43)$ in home and $(0.77, 0.69, 0.63, 0.59)$ in the foreign country. Trade liberalization improves the expected profit of exporting firms. Since the firms have better expectation on the marginal value of a vacancy, they create more vacancies, which increases the v/u ratios following the vacancy creation condition in (13).

However, the v/u ratio of the sector with comparative disadvantage in each country moves in the opposite direction. Upon trade liberalization, the firms in the importing sector have mixed expectation in their long-run perspectives. They are exposed to more serious competitive pressure from the foreign exporting firms, but the aggregate income effect strengthens the domestic demand in the long-run. As we see in [Table 2], the latter dominates the former in the foreign country, while it is dominated by the former in home country. It affects the v/u ratio and vacancy creation decision in the opposite direction.

Trade liberalization encourages sectoral migration of workers since it fuels the value differential among the unemployed. Note that the life time value is higher in the sector with comparative advantage due to faster wage growth. At the same time, there are more vacancies in the sector. The steady state unemployment is determined by the relative rates of inflows and outflows, which is not necessarily monotone. In our model, unemployment rate in the exporting sector are $(5.21, 5.17, 5.12, 5.06)$ in the home county and $(4.78, 4.91, 4.99, 5.04)$ in the foreign country under the respective tariff rates of $(0.02, 0.03, 0.04, 0.05)$.

Welfare We find that trade liberalization drives severe consumption inequality (or value differential) across sector. The workers employed in the exporting sector consumes more in spite of a higher price of final goods because their income goes up further in both countries. The unemployed worker in the sector also enjoy a higher lifetime value due to a higher job perspective. The workers in the importing sector consumes less than before. This value differential in workers in different sector implies a welfare cost from trade liberalization.

3.2.2 With Reverse Migration

We simulate our model by relaxing the assumption on one-side rigidity in labor mobility and allowing bilateral labor mobility. [Table 3] reports how the effects of trade liberalization vary upon this parameter change. In general, we find the qualitatively consistent effects of trade liberalization on steady-state outcome as shown in the benchmark model. The mutual tariff cut raises production efficiency by driving each country to specialize according to comparative advantage and efficiently reallocate resources. We also find similar labor market behaviors of steady-state vacancy, unemployment, and v/u ratio. It allows us to focus on the migration flow from the comparative disadvantage sector to comparative advantage sector.

4 Transition Dynamics

4.1 Characterization of Transition Dynamics

In this section, we analyze the transition dynamics to the new steady state after mutual tariff cut. As we mention in the definition of our equilibrium, the entire system is governed by the system of differential equations with initial values. Suppose that $\{J^h_{i0}, \tilde{J}^h_{i0}, V_{i0}, \tilde{V}_{i0}, H_{i0}, \tilde{H}_{i0}, u_{i0}, \tilde{u}_{i0}\}_{i=1,2}$ are given. We get $\{\theta_{i0}, \tilde{\theta}_{i0}\}_{i=1,2}$ from the job creation decision in (13) and $\{v_{i0}, \tilde{v}_{i0}\}_{i=1,2}$ from the definition of the market tightness parameters. In addition, by plugging $\{V_{i0}, \tilde{V}_{i0}\}_{i=1,2}$ into the definition of $\{\omega_{i0}, \tilde{\omega}_{i0}\}_{i=1,2}$, we immediately obtain the value of them. Once the price vector $(p_{10}, p_{20}, \tilde{p}_{10}, \tilde{p}_{20})$ is given such that $(p_{10}, p_{20}) = ((1+\tau)\tilde{p}_{10}, \tilde{p}_{20}/(1+\tilde{\tau}))$, the capital market clearing condition described in (19) and the feasibility constraint in the capital market $K = M_1 k_{1t} + M_2 k_{2t}$ jointly determine $\{\gamma_{i0}, \tilde{\gamma}_{i0}, k_{i0}, \tilde{k}_{i0}\}_{i=1,2}$. Wages $\{w_{i0}, \tilde{w}_{i0}\}_{i=1,2}$ are also obtained from the wage equation in (18) and $\{\pi_{i0}, \tilde{\pi}_{i0}\}_{i0}$ from (14). Then, we can calculate the demand and supply of final goods. By equating the demand and supply in each market, we obtain a price vector of (p_{10}, p_{20}), which should be consistent with

Table 3: Steady State Outcomes with Reverse Migration

	Home		Foreign	
	labor intensive Sector	capital intensive Sector	labor intensive Sector	capital intensive Sector
		Panel A: $\tau = \tilde{\tau} = 0.05$		
y_i	0.239	1.215	0.658	0.721
$\sum y_i$	1.454		1.379	
χ_i	-	0.161	0.229	-
k_i	0.230	1.770	0.354	0.646
h_i	0.245	0.691	0.996	0.849
p_i	1.636	1.270	1.558	1.333
w_i	1.386	1.663	0.888	0.848
γ_i	0.211	0.211	0.358	0.358
v_i	0.012	0.082	0.060	0.041
u_i	0.022	0.043	0.080	0.074
θ_i	0.539	1.930	0.744	0.553
u/L	0.064		0.077	
E_i	55.836	65.202	34.433	33.135
V_i	55.498	64.354	34.006	32.790
EV	62.663		33.804	
		Panel B: $\tau = \tilde{\tau} = 0.04$		
y_i	0.221	1.234	0.675	0.707
$\sum y_i$	1.455		1.382	
χ_i	-	0.184	0.263	-
k_i	0.210	1.790	0.366	0.634
h_i	0.229	0.707	1.015	0.831
p_i	1.675	1.312	1.611	1.365
w_i	1.410	1.718	0.926	0.874
γ_i	0.216	0.216	0.360	0.360
v_i	0.010	0.085	0.063	0.040
u_i	0.021	0.043	0.081	0.073
θ_i	0.457	1.953	0.786	0.545
u/L	0.065		0.077	
E_i	55.269	65.348	34.902	33.266
V_i	54.968	64.493	34.458	32.925
EV	62.785		34.132	

Table 3: Steady State Outcomes with Reverse Migration (Contd.)

	Home		Foreign	
	labor intensive Sector	capital intensive Sector	labor intensive Sector	capital intensive Sector
	Panel C: $\tau = \tilde{\tau} = 0.03$			
y_i	0.201	1.257	0.690	0.694
$\sum y_i$	1.457		1.384	
χ_i	-	0.207	0.296	-
k_i	0.187	1.813	0.376	0.624
h_i	0.211	0.725	1.033	0.816
p_i	1.783	1.407	1.731	1.449
w_i	1.494	1.852	1.007	0.941
γ_i	0.223	0.223	0.367	0.367
v_i	0.008	0.088	0.068	0.040
u_i	0.021	0.044	0.080	0.071
θ_i	0.377	2.000	0.842	0.555
u/L	0.065		0.076	
E_i	54.906	65.847	35.509	33.562
V_i	54.644	64.977	35.043	33.216
EV	63.276		34.614	
	Panel D: $\tau = \tilde{\tau} = 0.02$			
y_i	0.177	1.282	0.701	0.685
$\sum y_i$	1.459		1.386	
χ_i	-	0.229	0.330	-
k_i	0.161	1.839	0.385	0.615
h_i	0.188	0.747	1.047	0.805
p_i	2.016	1.597	1.976	1.629
w_i	1.685	2.133	1.167	1.083
γ_i	0.235	0.235	0.382	0.382
v_i	0.006	0.094	0.073	0.041
u_i	0.020	0.045	0.079	0.069
θ_i	0.294	2.078	0.919	0.592
u/L	0.065		0.074	
E_i	54.830	66.861	36.330	34.116
V_i	54.611	65.966	35.833	33.753
EV	64.314		35.330	

the initial price vector. At each $t \in [0, \infty)$, the bargaining rule in (16) and the job creation decision (13) determines $E_{it} - V_{it} = \frac{\phi_i \eta}{(1-\phi_i)q(\theta_{it})}$. Plugging it into (5) yields

$$\dot{V}_{it} \;=\; (r+\rho)V_{it} - bP_t^{-1} - \frac{\phi_i \eta \theta_{it}}{1-\phi_i} - \mu_i \omega_{it}(V_{i't} - V_{it}). \tag{43}$$

By doing this, we can substitute out $\{E_{it}, \tilde{E}_{it}\}_{i=1,2}$ from the system. Then, we recursively solve for the evolution of values $\{J_{it}^h, \tilde{J}_{it}^h, V_{it}, \tilde{V}_{it}\}_{i=1,2}$ using (15) and (43), as well as the evolution of measures $\{H_{it}, \tilde{H}_{it}, u_{it}, \tilde{u}_{it}\}_{i=1,2}$ using (23) and (24). The whole transition path is pinned down to an initial value problem(IVP), a system of differential equations with initial values.

The challenging point is that the values $\{J_{it}^h, \tilde{J}_{it}^h, V_{it}, \tilde{V}_{it}\}_{i=1,2}$ are forward-looking variables. With consistent initial values, the world economy converges to the new steady state.[13] In particular, the initial values of $\{J_{i0}^h, \tilde{J}_{i0}^h\}_{i=1,2}$ generate completely different dynamics right after the mutual tariff cut. Firms can immediately downsize with a discrete downward jump in their size, they are not able to hire a mass of workers immediately due to search friction.[14] Depending on firms' expectation, we have three different cases at time zero as follows.

[Case 1: $J_{i0}^h > J_i^h$] If the mutual tariff cut increases the demand for good i in the world market afterwards, the prices received by firms clearly rise up, which increase the marginal value of labor J_{i0}^h. Since firms are not able to hire more workers immediately, they create more vacancies.

$$H_{i0} = H_i, \quad u_{i0} = u_i, \quad \theta_{i0} > \theta_i, \quad \text{and} \quad v_{i0} > v_i.$$

[Case 2: $J_{i0}^h \in (0, J_i^h]$] The mutual tariff cut may lower the demand for good i but not too much. Although firms in sector i are required to downsize, they actually do by reducing new hiring rather than firing. Then,

$$H_{i0} = H_i, \quad u_{i0} = u_i, \quad \theta_{i0} \in (0, \theta_i), \quad \text{and} \quad v_{i0} \in (0, v_i).$$

[Case 3: $J_{i0}^h < 0$] If the (world) demand for good i are expected to shrink dramatically afterwards, firms in sector i want to reduce their employment immediately and create no vacancies. Employed workers are laid off. The firms choose $h_{i0'} < h_{i0}$ such that $J_{i'0}^h(h_{i'0}) = 0$.

$$H_{i0} < H_i, \quad u_{i0} > u_i, \quad \theta_{i0} = 0, \quad \text{and} \quad v_{i0} = 0.$$

Depending on its expectation, each sector in each country moves in a different direction. The numerical algorithm to solve the transition dynamics is postponed in Appendix B.

[13] We couldn't prove the existence, uniqueness, and convergence of the steady state. Instead, we will check it through numerical experiments.

[14] The asymmetric response in the frictional labor market prevents the world economy from jumping to the new steady state immediately. It is pointed out by Mortensen and Pissarides (1994) for the first time. Instead, in our model, firms revise their employment decision on capital and their exporting decision in response to the demand changes in home and foreign markets.

4.2 Simulation Results: Transition Dynamics

Output, Unemployment, and Consumption Inequality Figure 1 presents the transitory behaviors of key variables across countries and sectors. The red dotted line represents the level of variable prior to trade liberalization and the black dotted line represents the steady-state level after trade liberalization. The blue solid line shows the transition path of variable.

As we see in the previous section, trade liberalization increases the long-run output in both countries. In our simulation, total production, starting from 1.459 and 1.393 in the home and foreign country, converges to 1.457 and 1.400, respectively. Roughly, it takes 230 quarters at the 10^{-4} tolerance level. Interestingly, in the home country, trade liberalization may induce an immediate drop of total production quantity. There are two contrasting effects on output in each sector. The specialization process enforces the economic resources to be reallocated to the sector with comparative advantage, which means the output level in labor intensive sector monotonically decreases whereas the output level in capital intensive sector increases.[15] In the short-run, we find that output measure in the home country drops from 1.459 to 1.457 and takes 12 quarters (or 3years) to recoup the initial level. In the foreign country, we observe a quick peak and trough which is caused by a quick adjustment by capital.

This potential welfare loss in the transition is also observed in unemployment rate. Although trade liberalization has a positive on long-run employment, the short-run behavior is different from the long-run prediction. In the labor intensive sector, unemployment rate shows an immediate increase from 1.68% to 2.17% and it peaks on 3 quarters after the liberalization. Then, it slowly converges to steady state level of 1.48% after roughly 450 quarters. On the other hand, in the capital intensive sector or exporting sector, unemployment rate falls from 5.06% to 4.87% in the short-run in response to the fast growth of vacancies. As unemployed workers in the importing sector move onto the exporting sector, the unemployment in the exporting sector gradually increases to the convergence point.

Lastly, we also find that trade liberalization amplifies the inequality in terms of indirect utility flow across groups of workers. Figure 1 presents the variance of indirect utility across the sector and the employment status (employed or unemployed) as a proxy of consumption inequality. In both home and foreign country, the variance of indirect utility surges in the short-run and converges to the steady state which is higher than the initial level. This means that the unemployed workers suffer from inequality more seriously in the short-run. Interestingly, the foreign country with abundant labor resource shows long-run gains in both output and unemployment measures without incurring a considerable short-run cost in all three measures. However, the home country incurs the short-run welfare costs due to the rigidity in workers' mobility in the process of specializing a comparative advantage sector.

Input and Output Market Variables We present the simulation results on the transition path of the price of final goods and each factor price in Figure 4, 5, and 6 in Appendix C. All price variables show similar transitory behaviors in response to the trade liberalization. When the mutual tax cut is applied, the variables soar immediately and converge downward to the steady-state levels. This is because the

[15] The sector-level transition paths are illustrated in Appendix C.

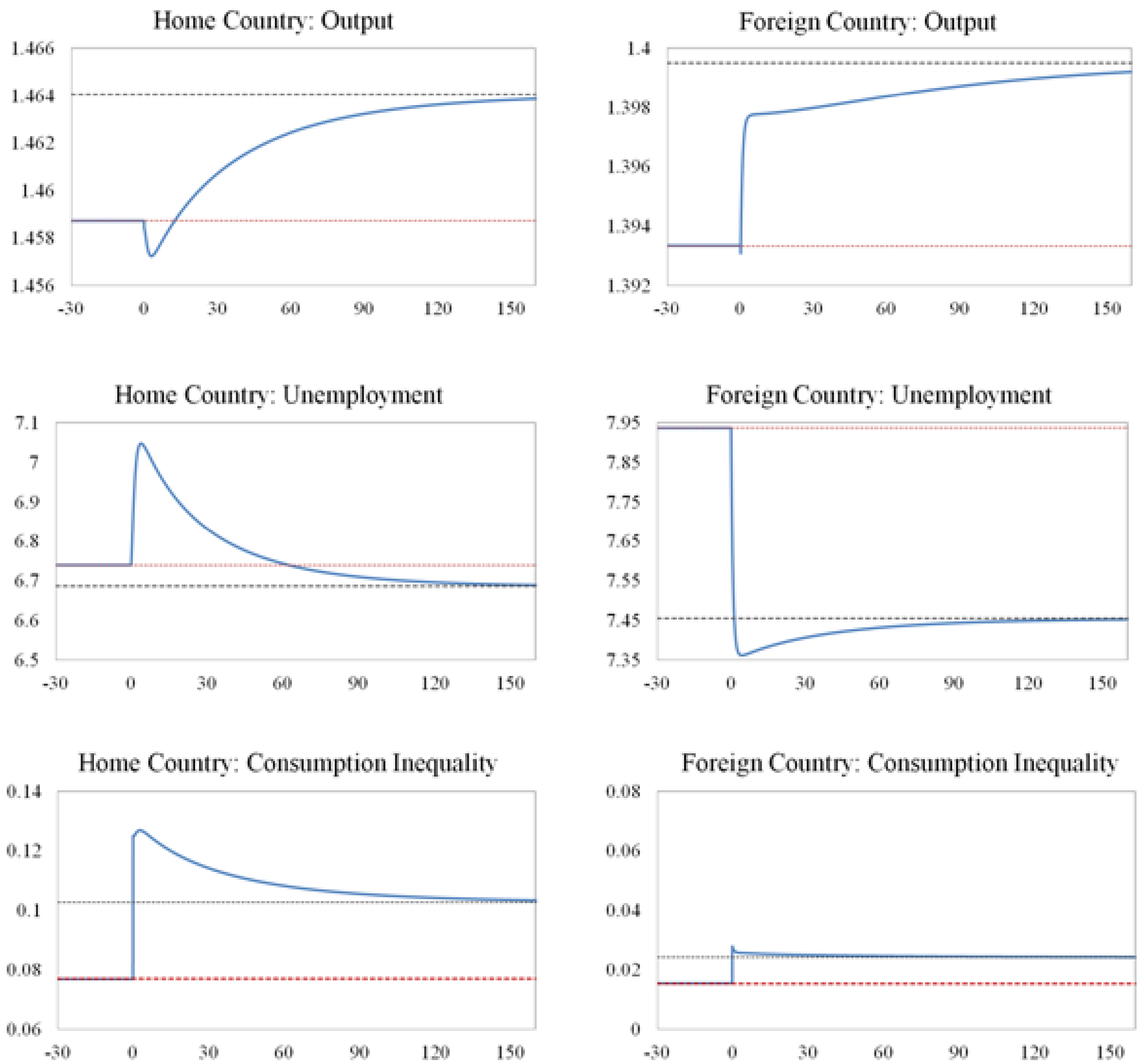

Figure 1: Key Variables in Transition

trade liberalization raises wages and the price of capital by increasing the marginal productivity gain from hiring workers and utilizing capital. (Firms create and maintain more job vacancy as well.) Consequently, the increase in wages and higher employment induces aggregate income effect which raises the price of final goods.

As shown in Figure 7, simulation results show that v/u ratio decreases only in the comparative disadvantage sector of the home country. As firms create less job vacancy (v decreases), more workers are unemployed and transferred to the comparative advantage sector (u increases). On the other hand, the comparative advantage sector in the home country and both sectors in foreign country show immediate increases of v/u ratio in the short-run and downward converging paths to the steady state level. In these sectors, firms create more job vacancy (v increases) and more unemployed workers are hired even after absorbing transferred unemployed workers from the sector with comparative disadvantage (u decreases).

5 Discussion: The Grace Period

Suppose that both governments mutually agree that they would keep $(\tau^1, \tilde{\tau}^1)$ in the first $\hat{T}$ years and set $(\tau^2, \tilde{\tau}^2)$ afterwards. That is,

$$(\tau_t, \tilde{\tau}_t) = \begin{cases} \tau^1, \tilde{\tau}^1 & \text{if } \ t < \hat{T} \\ \tau^1, \tilde{\tau}^1 & \text{if } \ t > \hat{T}. \end{cases} \tag{44}$$

In fact, it is suboptimal to introduce a socially optimal grace period. However, the more flexible tariff schedule requires higher implementation cost.[16] Given that the grace periods are commonly used in reality, we focus on the grace period dynamics.

Trade Equilibrium with a flexible tariff schedule A *trade equilibrium with a flexible tariff schedule* consists of bounded time series of the tariff schedule $(\tau_t, \tilde{\tau}_t)$, choice rules $\{c_{it}, \tilde{c}_{it}, k_{it}, \tilde{k}_{it}, v_{it}, \tilde{v}_{it}\}_{i=1,2}$, labor market tightness parameters $\{\theta_{it}, \tilde{\theta}_{it}\}_{i=1,2}$, price vector $\{p_{it}, \tilde{p}_{it}, \gamma_{it}, \tilde{\gamma}_{it}, w_{it}, \tilde{w}_{it}\}$, profit flow $\{\pi_{it}, \tilde{\pi}_{it}\}_{i=1,2}$, value equations $\{E_{it}, \tilde{E}_{it}, V_{it}, \tilde{V}_{it}, J_{it}^h, \tilde{J}_{it}^h\}_{i=1,2}$, and measures $\{H_{it}, \tilde{H}_{it}, h_{it}, \tilde{h}_{it}, u_{it}, \tilde{u}_{it}\}_{i=1,2}$ at every $t \in [0, \infty)$ such that:

(i) Each household in home (foreign) country optimally chooses $\{c_{1t}, c_{2t}\}$ $(\{\tilde{c}_{1t}, \tilde{c}_{2t}\})$ at every t.

(ii) Each firm in sector i in home (foreign) country optimally chooses $\{k_{it}, v_{it}\}$ $(\{k_{it}, v_{it}\})$ at every t. It also determines $\{\pi_{it}, \tilde{\pi}_{it}\}_{i=1,2}$ at every t.

(iii) The aggregate consistency requires that the vacancy creation decision by the individual firm based on (13) should be consistent with the definition of market tightness $\{\theta_{it}, \tilde{\theta}_{it}\}_{i=1,2}$ in (20) at every t.

(iv) The world market clearing conditions in (31) and (34) and the wage setting rule in (18) jointly determine $\{p_{it}, \tilde{p}_{it}, \gamma_{it}, \tilde{\gamma}_{it}, w_{it}, \tilde{w}_{it}\}_{i=1,2}$ at every t. By construction, $p_{1t} = (1+\tau)\tilde{p}_{1t}$, $(1+\tilde{\tau})p_{2t} = \tilde{p}_{2t}$, $\gamma_{1t} = \gamma_{2t}$, and $\tilde{\gamma}_{1t} = \tilde{\gamma}_{2t}$ at every t.

(v) The evolution of the entire system is recursively governed by the law of motion of (5), (6), (15), (23), and (24) given $\{E_{i0}, \tilde{E}_{i0}, V_{i0}, \tilde{V}_{i0}, J_{i0}^h, \tilde{J}_{i0}^h\}_{i=1,2}$ and $\{H_{i0}, \tilde{H}_{i0}, u_{i0}, \tilde{u}_{i0}\}_{i=1,2}$.[17]

6 Conclusion

In spite of its significance, the structural unemployment associated with trade liberalization and the implied welfare cost has not been properly treated in the traditional trade theory. Recently, some papers attempt to combine the labor search model with the international trade framework and analyze the long-run equilibrium unemployment under monopolistic competition. In contrast, by incorporating search and matching friction into two-factor, two-sector, two-country Heckscher-Ohlin framework, we develop a new dynamic general equilibrium model emphasizing comparative advantage

[16] Alternatively, one can think of a completely flexible tariff schedule following the unemployment insurance literature such as Hopenhayn and Nicolini (1997).

[17] By the law of motion in (6), one can easily find out that $H_{it} = M_i h_{it}$ at every t.

to analyze the entire dynamic path from the original steady state to the new steady state after trade reform.

Our numerical experiments show that trade liberalization improves firms' expected profit and gives them incentives to create more vacancies and hire more workers. The higher level of employment increases the aggregate income and shifts out the aggregate demands for consumption goods in the long run, which raises the both input and output prices. The higher levels of employment and wages reinforce and is reinforced by the aggregate demands and so on. The employment effect and the implied long-run real inflation increase consumption inequality across employment status, across sectors.

When the tariff is removed or reduced, firms on comparative advantage sectors create more vacancies in prospect of a higher profit, while firms on comparative disadvantage sectors reduce their size. The employment decision by the forward-looking firms immediately follows from trade reform. However, the sectoral migration decision is also affected by other issues such as specific human capital, culture, institution, preference, and so on. The discrepancy between the worker flow and vacancy flow causes labor market congestion, which makes employment and welfare fluctuating in the short-run transition path and also prolong the transition path itself. How painful the transition is and how long it is depends on the size of the initial unemployment pool as long as the primary source of welfare cost is labor market congestion due to the discrepancy between vacancy creation and worker flow. The country with a large size of it can effectively absorb the short-run impact of the trade liberalization.

Sometimes policy makers attempt to evaluate the effect of the tariff cut by comparing steady state outcomes. We should check first how long it takes to the steady state and how painful the transition is. We suggest a simple but general dynamic trade model with a grace period. We hope to pursue it further as a future research.

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Appendices

A The Optimal Control by Individual Firms

Consider the optimal control problem of the firm in sector i at time t. The firm chooses (k_{is}, v_{is}) at every $s \in [t, \infty)$ to maximize

$$\int_t^\infty e^{-r(s-t)}[R_{is} - \eta v_{is}]ds \tag{A1}$$

subject to

$$\begin{aligned} \dot{h}_{is} &= -(\delta+\rho)h_{is} + q(\theta_{is})v_{is} && \text{(A2)} \\ h_{it} &= \overline{h}_i && \text{(A3)} \end{aligned}$$

Simply, we ignore the restriction on the domain and solve for the optimal control problem. Then, we will check whether the interior solution is obtained. The Hamiltonian for the above problem is

$$\mathcal{H} = e^{-r(s-t)}[R_{is} - \eta v_{is}] - \pi_h[(\delta+\rho)h_{is} - q(\theta_{is})v_{is}].$$

The maximum principle implies that

$$\begin{aligned} k_{is} &: \gamma_i = (1-\beta_i)p_{is}\alpha_i k_{is}^{-\beta_i}h_{is}^{\beta_i} - \frac{\partial w_{is}}{\partial k_i}h_{is} && \text{(A4)} \\ v_{is} &: \eta e^{-r(s-t)} = \pi_h q(\theta_{is}) && \text{(A5)} \\ h_{is} &: \dot{\pi}_h = -e^{-r(s-t)}\frac{\partial R_{is}}{\partial h_{is}} + \pi_h(\delta+\rho) && \text{(A6)} \end{aligned}$$

From (A6),

$$\begin{aligned} & e^{-(\delta+\rho)(s-t)}\dot{\pi}_h - (\delta+\rho)e^{-(\delta+\rho)(s-t)}\pi_h = -e^{-(r+\delta+\rho)(s-t)}\frac{\partial R_{is}}{\partial h_{is}} \\ \iff\ & e^{-(\delta+\rho)(s-t)}\pi_h = \int_s^\infty e^{-(r+\delta+\rho)(\tau-t)}\frac{\partial R_{i\tau}}{\partial h_{i\tau}}d\tau + A_{ih} \\ \iff\ & \pi_h = e^{(\delta+\rho)(s-t)}\int_s^\infty e^{-(r+\delta+\rho)(\tau-t)}\frac{\partial R_{i\tau}}{\partial h_{i\tau}}d\tau + A_{ih}e^{(\delta+\rho)(s-t)} \end{aligned}$$

Since the shadow price π_h cannot diverge as $s \to \infty$, $A_{ih} = 0$. Thus, we get

$$\pi_h = e^{(\delta+\rho)(s-t)}\int_s^\infty e^{-(r+\delta+\rho)(\tau-t)}\frac{\partial R_{i\tau}}{\partial h_{i\tau}}d\tau \tag{A7}$$

Plugging (A7) into (A5) and rewriting yields

$$\eta = q(\theta_{is})\int_s^\infty e^{-(r+\delta+\rho)(\tau-s)}\frac{\partial R_{i\tau}}{\partial h_{i\tau}}d\tau \tag{A8}$$

Taking derivative of the objective function with respect to h_i yields

$$\frac{\partial J_{it}}{\partial h_i} = \int_t^\infty e^{-(r+\delta+\rho)(s-t)}\frac{\partial R^*_{is}}{\partial h_{is}}ds \tag{A9}$$

Connecting (A8) and (A9) results in

$$\eta = q(\theta_{is})\frac{\partial J_{is}}{\partial h_i} \tag{A10}$$

Let $J^h_{it} := \frac{\partial J_{it}}{\partial h_i}$. From (A9),

$$\begin{aligned}
J^h_{it} &= \int_t^{t+dt} e^{-(r+\rho+\delta)(s-t)}\frac{\partial R^*_{is}}{\partial h_{is}}ds + e^{-(r+\rho+\delta)dt}J^h_{it+dt} \\
J^h_{it} - e^{-(r+\rho+\delta)dt}J^h_{it} &= \int_t^{t+dt} e^{-(r+\rho+\delta)(s-t)}\frac{\partial R^*_{is}}{\partial h_{is}}ds + e^{-(r+\rho+\delta)dt}(J^h_{it+dt} - J^h_{it}) \\
\frac{J^h_{it} - e^{-(r+\rho+\delta)dt}J^h_{it}}{dt} &= \frac{1}{dt}\int_t^{t+dt} e^{-(r+\rho+\delta)(s-t)}\frac{\partial R^*_{is}}{\partial h_{is}}ds + e^{-(r+\rho+\delta)dt}\frac{(J^h_{it+dt} - J^h_{it})}{dt} \\
(r+\rho+\delta)J^h_{it} &= \frac{\partial R^*_{it}}{\partial h_{it}} + \dot{J}^h_{it}
\end{aligned}$$

Then,

$$\dot{J}^h_{it} = (r+\rho+\delta)J^h_{it} - [p_{it}\frac{\partial y^*_{it}}{\partial h_{it}} - w_{it} - \frac{\partial w_{it}}{\partial h_{it}}h_{it}] \tag{A11}$$

$$\dot{E}_{it} = (r+\rho+\delta)E_{it} - w_{it}P_t^{-1} - \delta V_{it} \tag{A12}$$

$$\dot{V}_{it} = (r+\rho)V_{it} - bP_t^{-1} - f(\theta_{it})(E_{it} - V_{it}) - \omega_{it}(V_{i't} - V_{it}) \tag{A13}$$

Then, Stole and Zwiebel (1996) implies that

$$(1-\phi_i)(E_{it} - V_{it}) = \phi_i J^h_{it} \;\text{ for any }\; t \in [0,\infty), \tag{A14}$$

and

$$(1-\phi_i)(\dot{E}_{it} - \dot{V}_{it}) = \phi_i \dot{J}^h_{it}. \tag{A15}$$

Then, for each $t \in [0,\infty)$,

$$\begin{aligned}
&\phi_i[\frac{\partial y^*_{it}}{\partial h_{it}} - w_{it} - \frac{\partial w_{it}}{\partial h_{it}}h_{it}] \\
&= (1-\phi_i)[(w_{it}-b)P_t^{-1} - f(\theta_{it})(E_{it} - V_{it}) - \mu_i\omega_{it}(V_{i't} - V_{it})] \\
&= (1-\phi_i)[(w_{it}-b)P_t^{-1} - \frac{\phi_i f(\theta_{it})J^h_{it}}{1-\phi_i} - \mu_i\omega_{it}(V_{i't} - V_{it})] \\
&= (1-\phi_i)(w_{it}-b)P_t^{-1} - \eta\phi_i\theta_{it} - (1-\phi_i)\mu_i\omega_{it}(V_{i't} - V_{it})
\end{aligned}$$

Reordering yields

$$\begin{aligned}
[(1-\phi_i)P_t^{-1} + \phi_i]w_{it} + \frac{\partial w_{it}}{\partial h_{it}}\phi_i h_{it} &= \phi_i p_{it}\frac{\partial y^*_{it}}{\partial h_{it}} \\
&+(1-\phi_i)bP_t^{-1} + \eta\phi_i\theta_{it} + (1-\phi_i)\mu_i\omega_{it}(V_{i't} - V_{it})
\end{aligned}$$

Since it should be true for all $t \in [0,\infty)$, the solution of the differential equation in the above has the form of

$$w_{it} = B_{it}\frac{\partial y^*_{it}}{\partial h_{it}} + C_{it}, \tag{A16}$$

where neither B_{it} nor C_{it} depends on h_{it}. Plugging the expression into the above and applying the undetermined coefficient method yields

$$\begin{aligned} B_{it} &= \frac{\phi_i p_{it}}{(1-\phi_i)P_t^{-1} + \phi_i \beta_i}, \quad \text{and} \\ C_{it} &= \frac{bP_t^{-1} + \eta\theta_{it}\phi_i/(1-\phi_i) + \mu_i \omega_{it}(V_{i't} - V_{it})}{P_t^{-1} + \phi_i/(1-\phi_i)}. \end{aligned}$$

Pugging (A16) into (A4) and rewriting yields

$$\gamma_i = \frac{(1-\phi_i)P_t^{-1}}{(1-\phi_i)P_t^{-1} + \phi_i\beta_i}(1-\beta_i)p_{it}\alpha_i k_{it}^{-\beta_i} h_{it}^{\beta_i}. \tag{A17}$$

Since $\gamma_i = \gamma_{i'}$ and $k_{it} + k_{i'} = K$ on equilibrium, we get

$$\frac{(1-\phi_i)(1-\beta_i)p_{it}\alpha_i}{(1-\phi_i)P_t^{-1} + \phi_i\beta_i} \cdot \frac{h_{it}^{\beta_i}}{k_{it}^{\beta_i}} = \frac{(1-\phi_{i'})(1-\beta_{i'})p_{i't}\alpha_{i'}}{(1-\phi_{i'})P_t^{-1} + \phi_{i'}\beta_{i'}} \cdot \frac{h_{i't}^{\beta_{i'}}}{(K-k_{it})^{\beta_{i'}}} \tag{A18}$$

Given $(p_{it}, p_{i't}, h_{it}, h_{i't})$, equation (A18) determines $(k_{it}, k_{i't})$ and $\gamma_{it}(= \gamma_{i't})$.

B Numerical Experiments

B.1 Parameterization

A time period is normalized to one quarter and the discount rate is fixed at r = 0.012, which is consistent with the annual interest rate around 5%. In the benchmark case, we fix the tariff rate at 0.1 for both countries. The elasticity of substitution is fixed at 3.8 following Bernard, Eaton, Jensen, and Kortum (2003), Bernard, Redding, and Schott (2007) and Felbermayr, Prat, and Schmerer (2011). In particular, Bernard, Eaton, Jensen, and Kortum (2003) estimates σ using plant-level U.S. manufacturing data. Regarding the labor market, we follow Shimer (2005). He points out that the quarterly separation rate is 0.1 in U.S. labor market. Since his estimate includes both the retirement rate and separation rate, we impose $\rho + \delta = 0.1$. Then, we set $\delta = 0.09$ and $\rho = 0.01$, which allows workers to work for 25 years in expectation before her first retirement, and 80% of the workers (entering the labor market at their age 25) to retire before at their age 65. The value of (λ, κ, η) is directly borrowed from Shimer (2005). The efficiency and elasticity parameters of the matching function are given by $\lambda = 1.35$ and $\kappa = 0.72$, respectively. The vacancy creation cost η is fixed at 0.213. In Shimer (2005), these values are chosen to get the unemployment rate around 0.06 and job finding rate around 1.35 roughly. We calibrate the bargaining weight of the workers and unemployment benefit to ensure that the replacement ratio is between 30% and 40% of average wages and the average wages is around 1. This target results in $\phi_i = 0.77$ and $b = 0.3$. In production technology, we normalize $\alpha_i = 1$. Since we consider symmetric differences both in country factor endowments, $(L, \tilde{L}, K, \tilde{K}) = (1, 2, 2, 1)$, and in the factor intensities, we set $(\beta_1, \beta_2) = (0.7, 0.3)$. Finally, we assume that the sensitivity parameter of switching decision $\xi = 0.1$, which

Table 4: Baseline Parameterizations (Home)

Parameter	Interpretation	Value
r	interest rate	0.12
σ	elasticity of substitution parameter	3.8
ρ	retirement rate	0.01
δ	separation rate	0.09
κ	elasticity of matching function	0.72
λ	efficiency of matching function	1.35
η	vacancy creation cost	0.213
b	unemployment benefit	0.3
(ϕ_1, ϕ_2)	bargaining weight of worker	(0.77,0.77)
(α_1, α_2)	productivity parameter in each sector	(1.0, 1.0)
(β_1, β_2)	labor share in each sector	(0.6, 0.4)
μ	the arrival rate of revision shock	0.07
ξ	the sensitivity parameter of switching decision	0.1

ensures that the population share of the labor intensive sector 25% and the population share of the capital intensive sector is 75%. When we set $\xi = 0.2$, the share of the labor intensive sector is around 20%, and when we set $\xi = 0.05$, the share of the labor intensive sector is around 30% in home country. Without special notification, all parameters are the same across countries.

B.2 Computational Procedures: Steady State

In this subsection, we briefly explain the solution algorithm that we adopt to solve for the steady state equilibrium of our interest. Actually, there might be a lot of alternative algorithms. But we realize that this algorithm is quite fast.

1. Guess (p_1, p_2). Solve for P using (3).
   (a) Guess (h_1, h_2). Using

$$\begin{aligned} K &= M_1 k_1 + M_2 k_2 \quad \text{and} \\ \gamma &= \frac{\alpha_1(1-\beta_1)(1-\phi_1)p_1}{(1-\phi_1)+\phi_1\beta_1 P}\Big(\frac{h_1}{k_1}\Big)^{\beta_1} = \frac{\alpha_2(1-\beta_2)(1-\phi_2)p_2}{(1-\phi_2)+\phi_2\beta_2 P}\Big(\frac{h_2}{k_2}\Big)^{\beta_2}, \end{aligned}$$

   Calculate the values of (γ, k_1, k_2).
      i. Guess (V_1, V_2). Then, (ω_1, ω_2) immediately follows from its definition. Then, using

$$\Big(\frac{k_i}{h_i}\Big)^{1-\beta_i} = \frac{(1-\phi_i)+\phi_i\beta_i P}{\alpha_i\beta_i(1-\phi_i)p_i}\Big[\frac{\eta(r+\rho+\delta)}{q(\theta_i)} + \frac{bP^{-1} + \frac{\eta\phi_i\theta_i}{(1-\phi_i)} + \mu_i\omega_{i'}(V_{i'} - V_i)}{P^{-1} + \phi_i/(1-\phi_i)}\Big],$$

      solve for (θ_1, θ_2).

ii. Update (V_1, V_2) based on

$$rV_i = \nu(p_1^c, p_2^c, b) - \rho V_i + \frac{\phi_i \eta \theta_i}{1-\phi_i} + \mu_i \omega_{i'}(V_{i'} - V_i).$$

If the updated values, (V_1, V_2), are sufficiently close to the guessed values in step i, go to step (b); otherwise go back to step i with the updated (V_1, V_2).

(b) Calculate the invariant measures (H_1, H_2, u_1, u_2) using

$$\begin{aligned} 0 &= -(\rho+\delta)H_i + f(\theta_i)u_i, \quad \text{and} \\ 0 &= -(f(\theta_i) + \mu_i \omega_{i'} + \rho)u_i + \delta H_i + \mu_{i'}\omega_i u_{i'} + \rho \omega_i L. \end{aligned}$$

(c) Update (h_1, h_2) based on $h_i = H_i/M_i$. If the updated values (h_1, h_2) are sufficiently close to the guessed values in step (a), go to step 2; otherwise go back to step (a) with the updated values (h_1, h_2).

2. Solve for $(y_i, y_2 \pi_1, \pi_2, w_1, w_2)$ using

$$\begin{aligned} y_i &= \alpha_i k_i^{1-\beta_i} h_i^{\beta_i}, \\ \pi_i &= p_i y_i - \gamma k_i - w_i h_i - \eta v_i, \quad \text{and} \\ w_i &= \frac{\phi_i p_i (\partial y_i/\partial h_i)}{(1-\phi_i)P^{-1} + \phi_i \beta_{ih}} + \frac{bP^{-1} + \eta \phi_i \theta_i/(1-\phi_i) + \mu_i \omega_{i'}(V_{i'} - V_i)}{P^{-1} + \phi_i/(1-\phi_i)}. \end{aligned}$$

3. Let $(\tilde{p}_1, \tilde{p}_2) = (p_1/(1+\tau), (1+\tilde{\tau})p_2)$. Repeat step 1 and 2. with the foreign parameters and variables.

4. Calculate the excess demand for each sectoral goods using the market clearing conditions in (31) and (34). If the sum of both excess demands is sufficiently close to zero, stop; otherwise go to step 1.

B.3 Computational Procedures: Transition Dynamics

We simulate the transition path to the new steady state after trade reform by alternating the forward and backward shooting algorithms. Suppose the economy is in a particular steady state (not necessarily), which is described by 20 relevant variables: $(H_i, u_i, \tilde{H}_i, \tilde{u}_i, J_i, V_i, \tilde{J}_i, \tilde{V}_i, (k/h)_1, (\tilde{k}/\tilde{h})_1, p_i)$. At time 0, both countries agree on a mutual tariff cut. First, let us focus on the case where the impact of the tariff cut is small enough that no firms fire their workers along the transition path. (Case 1 and 2). In such cases, we keep the measures of workers at time 0 as

$$H_{i0} = H_i, \quad u_{i0} = u_i, \quad \tilde{H}_{i0} = \tilde{H}_i, \quad \text{and} \quad \tilde{u}_{i0} = \tilde{u}_i.$$

Assume that all variables converge to the new steady state after a sufficiently large amount of time (denoted by T). We know all values in the new steady state. However, we don't know the values of the forward-looking variables $(J_i^h, V_i, \tilde{J}_i^h, \tilde{V}_i)$ at time 0, because they can jump right after the mutual tariff cut agreement. Let

$$\vec{m}_t^{(j)} = (H_{1t}^{(j)}, H_{2t}^{(j)}, u_{1t}^{(j)}, u_{2t}^{(j)}, \tilde{H}_{1t}^{(j)}, \tilde{H}_{2t}^{(j)}, \tilde{u}_{1t}^{(j)}, \tilde{u}_{2t}^{(j)}),$$

$$
\begin{aligned}
\vec{v}^{(j)} &= (J_{1t}^{h(j)}, J_{2t}^{h(j)}, V_{1t}^{(j)}, V_{2t}^{(j)}, \tilde{J}_{1t}^{h(j)}, \tilde{J}_{2t}^{h(j)}, \tilde{V}_{1t}^{(j)}, \tilde{V}_{2t}^{(j)}), \quad \text{and} \\
\vec{p}^{(j)} &= ((k/h)_{1t}^{(j)}, (\tilde{k}/\tilde{h})_{1t}^{(j)}, p_{1t}^{(j)}, p_{2t}^{(j)}),
\end{aligned}
$$

where the superscript (j) indicates that the vector is obtained from jth iteration. We proceed as follows.

1. Set evenly spaced nodes $t_l = \frac{l}{2n}T$ $(l = 0, \ldots, 2n)$.
2. Guess the entire transition path of $(\vec{m}_t^{(0)}, \vec{v}_t^{(0)}, \vec{p}_t^{(0)})$ for every $t \in \{t_l\}_{l=0}^{2n}$. $\vec{m}_0^{(0)}$ should have the old steady state value, and $\vec{v}_T^{(0)}$ should do the new steady state value.
3. Repeat the following procedure until $(\vec{m}_t^{(j)}, \vec{v}_t^{(j)}, \vec{p}_t^{(j)})$ converge at each t.
   (a) Given $(\vec{v}_t^{(j-1)}, \vec{p}_t^{(j-1)})$, calculate new series $\hat{\vec{m}}_t$ by forward shooting. Then, update $\vec{m}_t^{(j)} = a\hat{\vec{m}}_t + (1-a)\vec{m}_t^{(j-1)}$ where $a \in (0,1)$. Since we work with RK4, we can solve the values only at the nodes with even number. Therefore, obtain the values at the nodes with odd number by interpolation.
   (b) Given $(\vec{m}_t^{(j)}, \vec{p}_t^{(j-1)})$, calculate new series $\hat{\vec{v}}_t$ by backward shooting. Update $\vec{v}_t^{(j)} = a\hat{\vec{v}}_t + (1-a)\vec{v}_t^{(j-1)}$ where $a \in (0,1)$. Obtain the values at the nodes with odd number by interpolation.
   (c) Given $(\vec{m}_t^{(j)}, \vec{v}_t^{(j)})$, update $\vec{p}_t^{(j)}$ by solving the clearing conditions in the capital market and the product market. Since we can solve the clearing conditions at every node, we don't need interpolation. Note that foreign price is obtained by $\tilde{p}_{1t} = p_{1t}/(1+\tau)$ and $\tilde{p}_{2t} = p_{2t}(1+\tilde{\tau})$. In case of the transition with a certain grace period, we denote by $(\tau^1, \tilde{\tau}^1)$ the tariff rates during the grace period and $(\tau^2, \tilde{\tau}^2)$ afterwards. Also, $((k/h)_{2t}, (\tilde{k}/\tilde{h})_{2t})$ is obtained by $H_{1t}(k/h)_{1t} + H_{2t}(k/h)_{2t} = K$ and $\tilde{H}_{1t}(\tilde{k}/\tilde{h})_{1t} + \tilde{H}_{2t}(\tilde{k}/\tilde{h})_{2t} = \tilde{K}$
      i. If the distance between $(\vec{m}_t^{(j-1)}, \vec{v}_t^{(j-1)})$ and $(\vec{m}_t^{(j)}, \vec{v}_t^{(j)})$ is large, it is inefficient to solve the clearing conditions simultaneously, in terms of both computation time and the stability of solution. Then, we solve and update each of them separately. Given $(\vec{m}_t^{(j)}, \vec{v}_t^{(j)}, (\tilde{k}/\tilde{h})_{1t}^{(j-1)}, p_{1t}^{(j-1)}, p_{2t}^{(j-1)})$, obtain $\widehat{(k/h)}_{1t}$ by solving the clearing condition in the capital market of home country. Update $(k/h)_{1t}^{(j)} = a\widehat{(k/h)}_{1t} + (1-a)(k/h)_{1t}^{(j-1)}$ where $a \in (0,1)$. Repeat the same procedure for $(\tilde{k}/\tilde{h})_{1t}$, p_{1t}, and p_{2t}.
      ii. If the distance is small enough, given $(\vec{m}_t^{(j)}, \vec{v}_t^{(j)})$, obtain $\hat{\vec{p}}_t$ by solving the clearing conditions simultaneously by the Newton-Raphson method. Update $\vec{p}_t^{(j)} = a\hat{\vec{p}}_t + (1-a)\vec{p}_t^{(j-1)}$ where $a \in (0,1)$.
4. If the distance between $(\vec{m}_t^{(j-1)}, \vec{v}_t^{(j-1)}, \vec{p}_t^{(j-1)})$ and $(\vec{m}_t^{(j)}, \vec{v}_t^{(j)}, \vec{p}_t^{(j)})$ at the predetermined node is sufficiently small, stop calculation and adopt $(\vec{m}_t^{(j)}, \vec{v}_t^{(j)}, \vec{p}_t^{(j)})$ as the solution.

Note that $\vec{m}_0^{(j)}$ and $\vec{v}_T^{(j)}$ are fixed at each jth iteration, since we apply shooting from these points. As long as the steady state and the path from the initial point to it is uniquely defined, $\vec{m}_T^{(j)}$ and $\vec{v}_0^{(j)}$ converge to the right values for a sufficiently large T.

C Simulation Results: Transition Dynamics

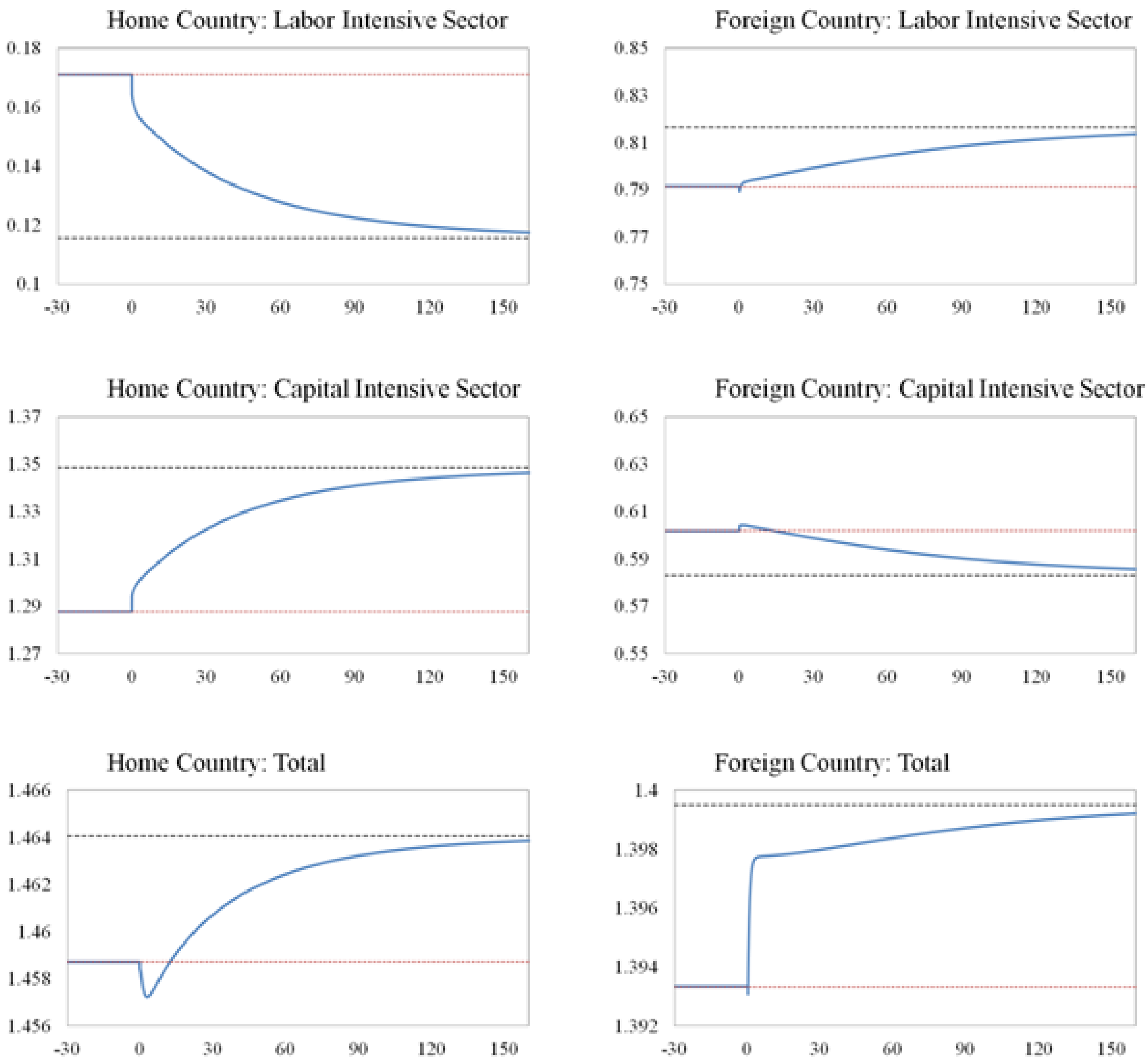

Figure 2: Output in Transition

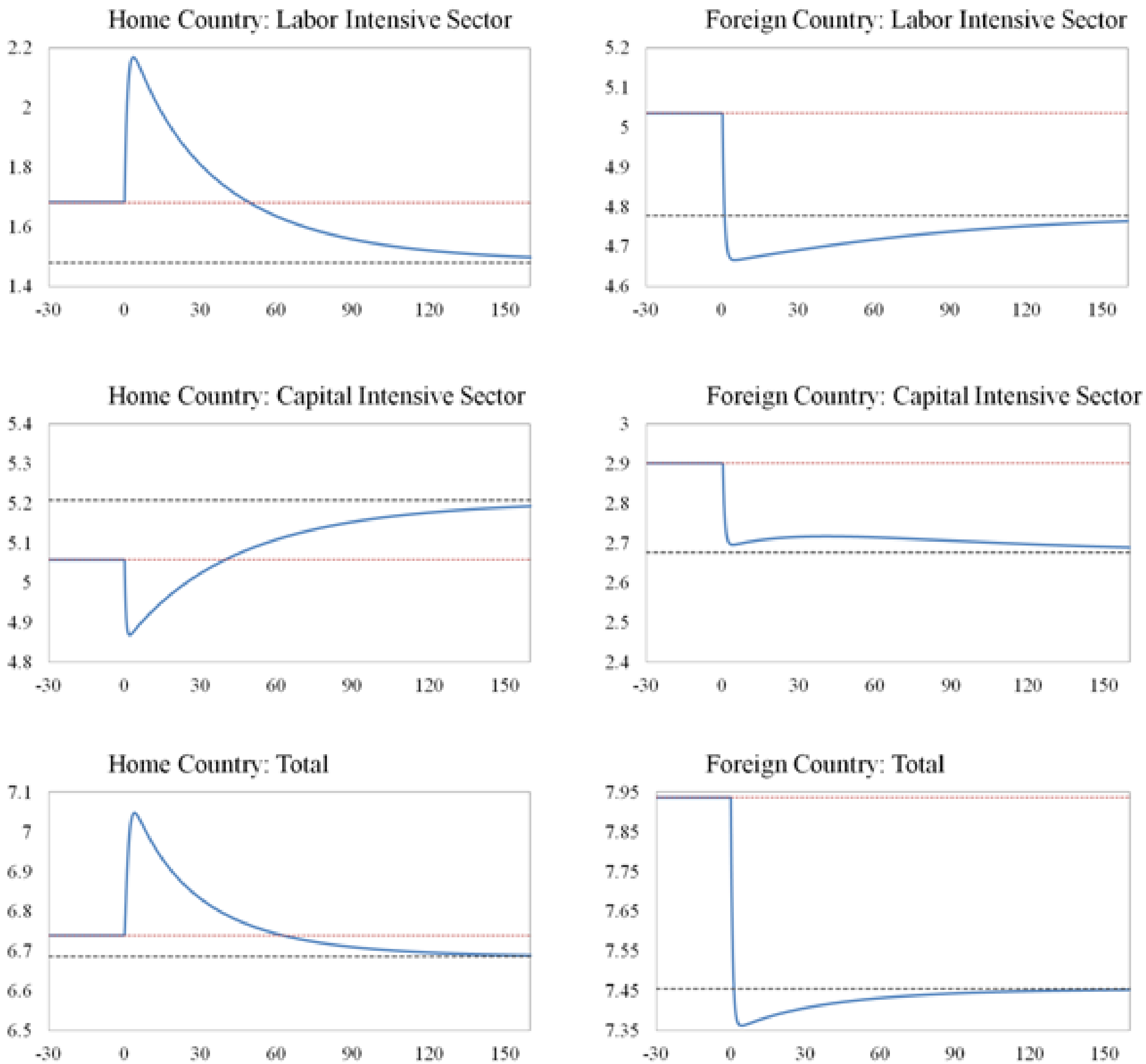

Figure 3: Unemployment Rates in Transition

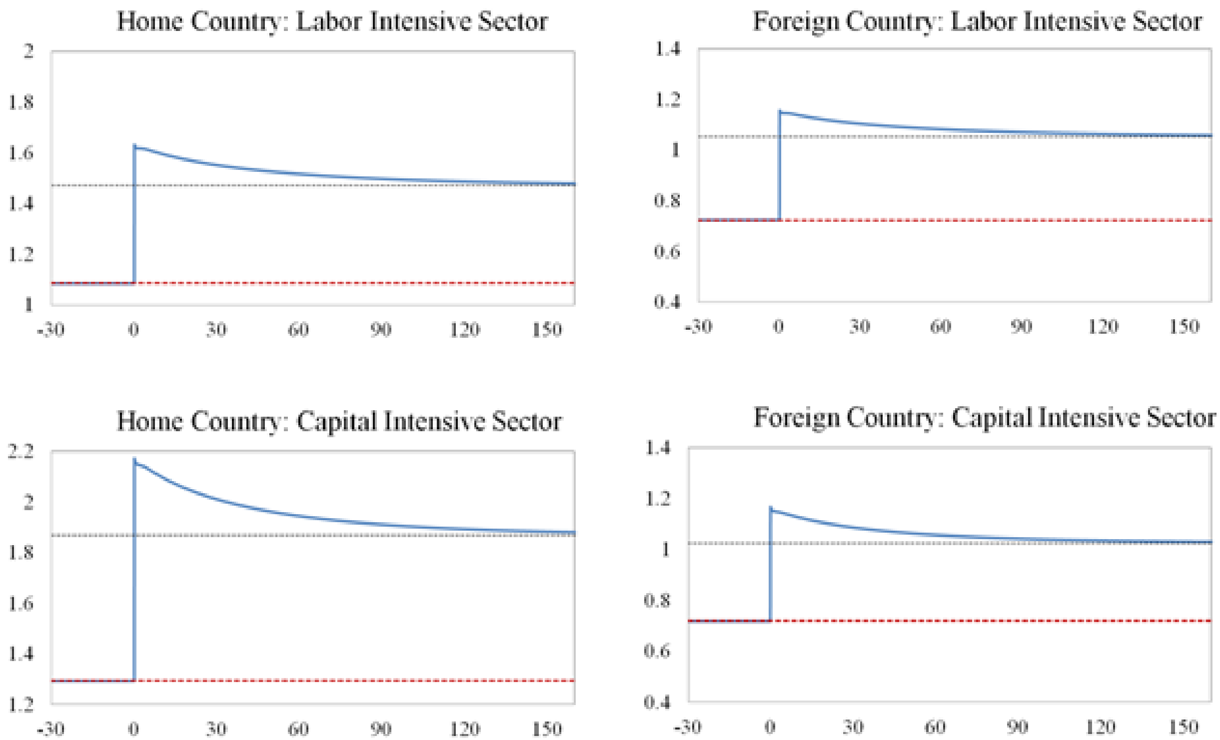

Figure 4: Wages in Transition

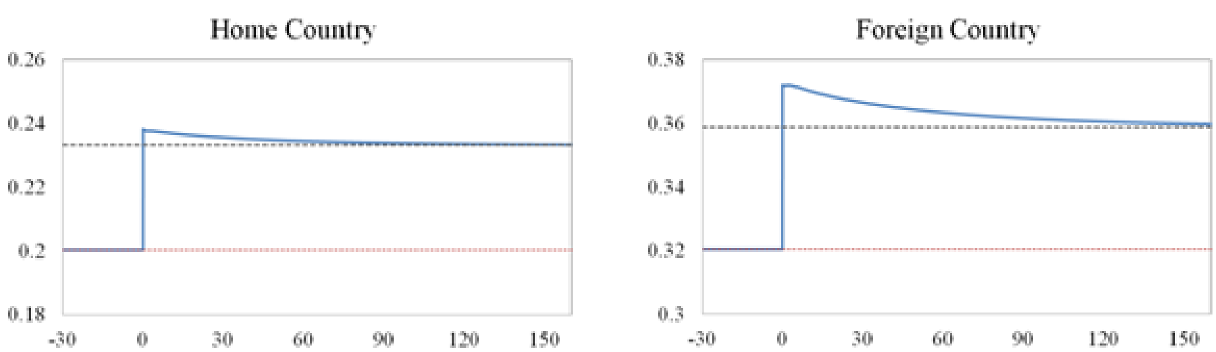

Figure 5: The Price of Capital in Transition

Figure 6: Prices in Transition

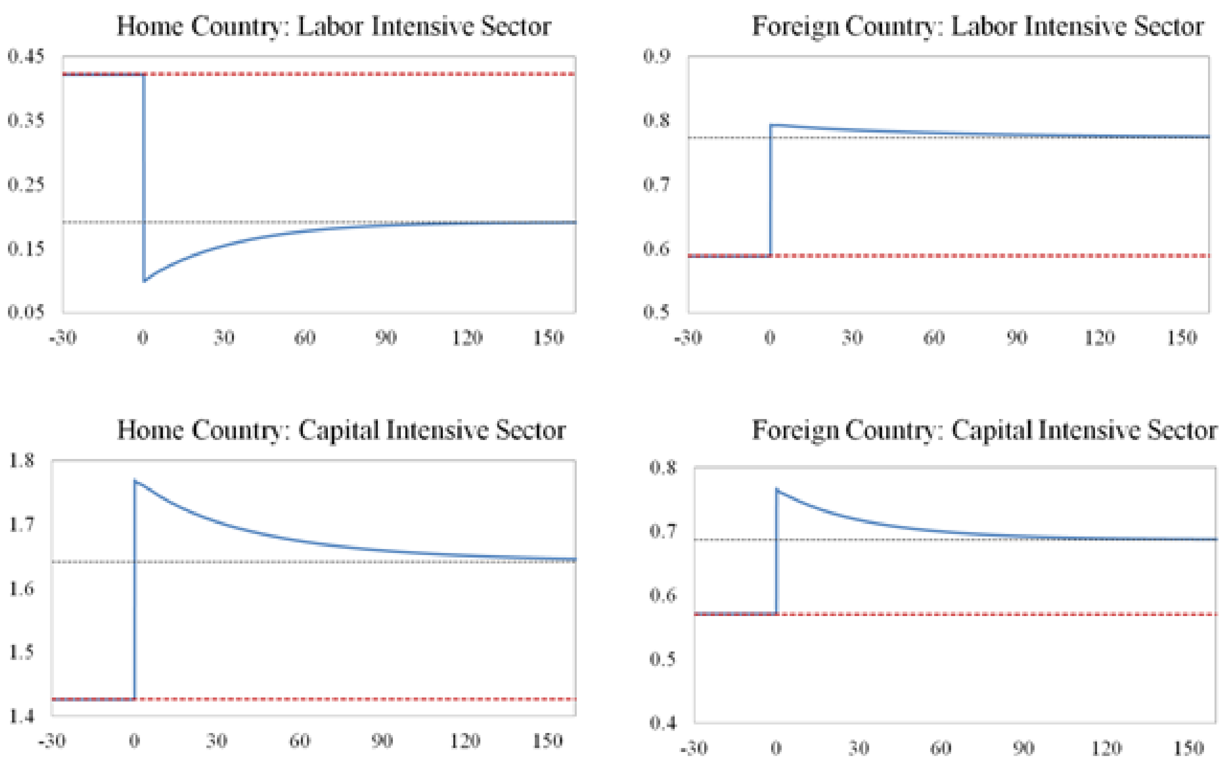

Figure 7: The Labor Market Tightness in Transition

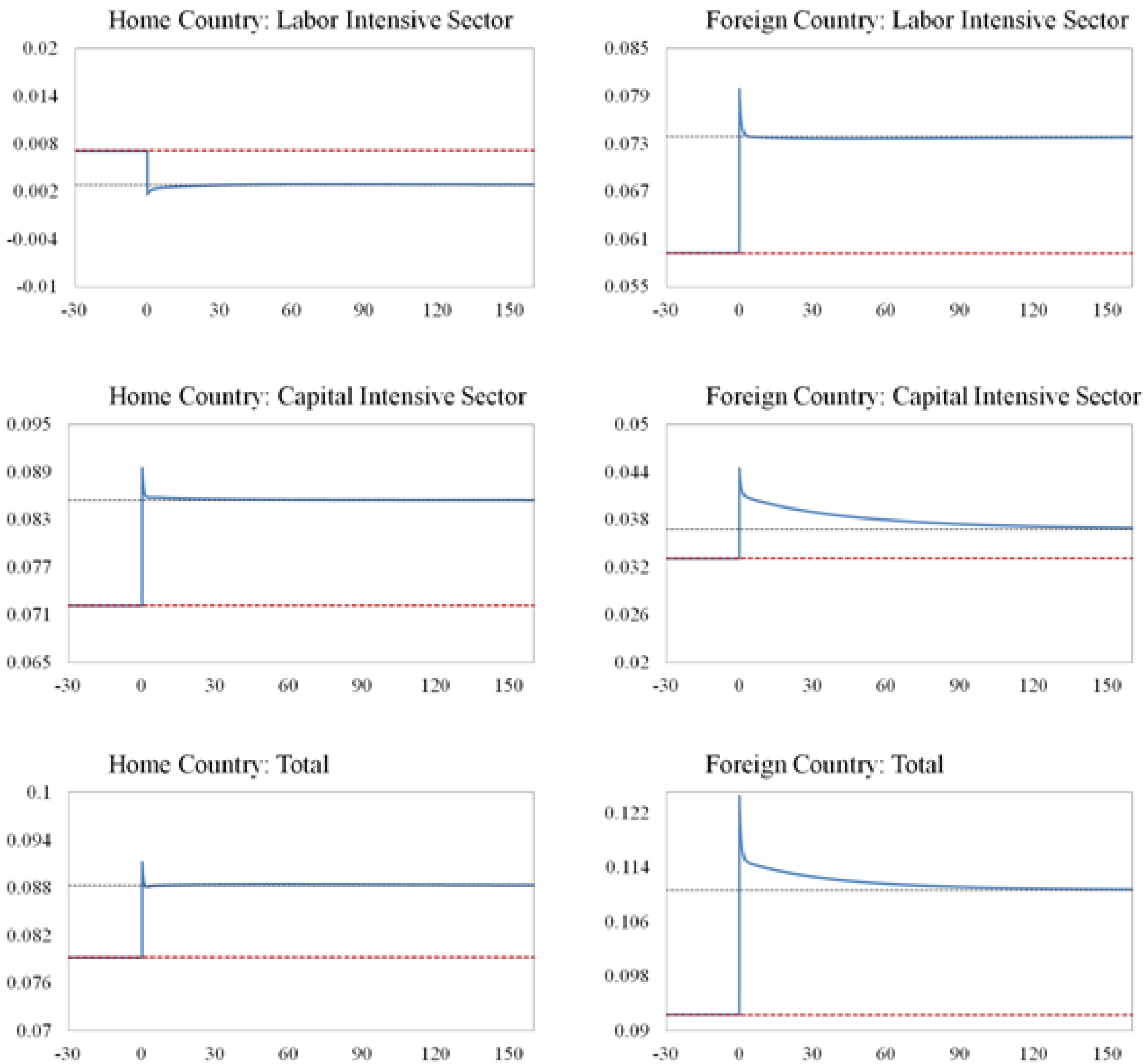

Figure 8: Vacancy Creation in Transition

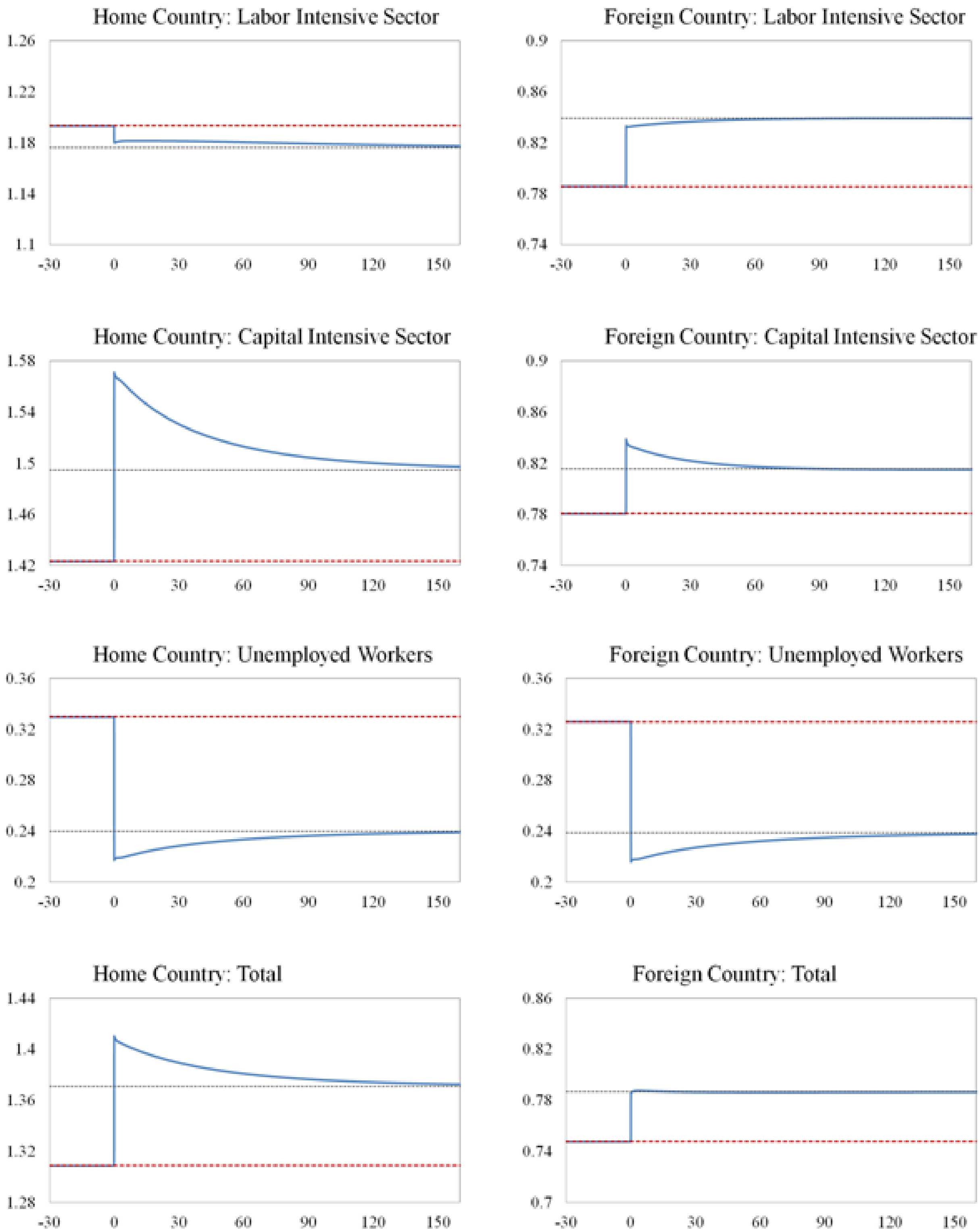

Figure 9: Indirect Utility in Transition