1. Introduction

The New Keynesian Phillips Curve (NKPC) has played an important role in recent theoretical work on inflation as well as monetary policy analysis. It explains the inflation dynamics through the relation of expected inflation and marginal cost, and its hybrid version includes lagged inflation as an additional component that shifts the curve. This paper empirically examines the US hybrid NKPC focusing on the roles of forward-looking and backward-looking components and studies the monetary policy implications. However, rather than using the traditional mean relations, we explore the relations in multiple quantiles. Analyzing multiple quantiles helps investigating various aspects of relations between inflation and the components in the NKPC other than the conditional mean. In general, those components may influence not only the conditional mean but also many other characteristics of the conditional distribution, such as expanding its dispersion, stretching one tail of the distribution, and even inducing multimodality. Thus, it is possible that the roles of backward-looking and forward-looking components in the NKPC vary across quantiles if they have assymmetric relations to the uncertainty structure of inflation, and relying only on the mean would not be sufficient to capture the relations. Explicit investigation of these relations via multiple quantile estimation can provide a more informative empirical analysis.

The primary purpose of using multiple quantile model in this paper is to make a useful inference of the asymmetric monetary policy. The asymmetry can be captured in two ways. First, the monetary authority may respond asymmetrically to different economic circumstances. If the central bank expects inflationary pressure in the near future, it has to focus more on the probability that future inflation will exceed a certain level, such as the target range in the inflation targeting system; thus, the equations for upper quantiles are more interesting. Second, the policy effect on the risk of inflation is asymmetric. If the quantile coefficients are asymmetric in the upper and lower quantiles, then the increase and decrease in the components change the distribution in an asymmetric way, indicating that the changes in the risk structure in expansionary and tightening monetary policies would be different.

The NKPC model used in this paper is the hybrid version. The canonical NKPC based on Calvo (1983) does not contain the lagged inflation term, and as Gali, Gertler, and López-Salido (2005) point out, the microfoundation of the lagged term is not clear. However, Fuhrer and Moore (1995) find that the canonical NKPC is not successful in explaining the stylized fact that the monetary policy has a delayed effect on inflation. Thus, many works augment the NKPC with lagged inflation to capture the persistency of inflation, calling it a hybrid NKPC, and thereby improve analysis of the lagged effect (Adam and Padula (2011), Fuhrer and Moore (1995), Gali and Gertler (1999)). On the other hand, Hall et al. (2009)

and Kim and Kim (2008) claim that a seemingly significant lagged inflation coefficient is primarily due to the misspecification of the NKPC such as the existence of a structural break and nonlinearity. Examining the backward looking component provides an important monetary policy implication. Without the lagged inflation, the monetary policy effect is generally described by the direct change in expected inflation due to the policy change. The presence of the backward looking component indicates the indirect effect of monetary policy by affecting the real economy, implying that the policy effects are more complicated.

We estimate the model using Chernozhukov and Hansen (2008)'s *instrumental variable quantile regression(IVQR)* and use Fitzenberger (1997)'s *moving blocks bootstrap* to estimate the heteroscedasticity and serial correlation robust (HAC) varianc-covariance matrix. Chortareas, Magonis, and Panagiotidis (2012) estimate a similar model with a different method using the Euro data. But their estimation is not based on the HAC estimation, which generally loses the consistency in this structural set-up of the NKPC. Also, their suggested estimation method does not provide the covariance estimators for coefficients in different quantiles, which makes it hard to test the asymmetry. Considering recent findings of the existence of structural breaks in the inflation process (Clark and McCracken (2006), Estrella and Fuhrer (2003), and Jouini and Boutahar (2003)), and the relations of the structural breaks and the backward-looking component (Kim and Kim (2008)), we also perform a test for a single structural break in each quantile level.

The estimation results without a structural break substantially differ across quantile levels. In upper quantiles, the estimated coefficients are close to those of canonical NKPC in that the expected inflation coefficients are higher and the lagged inflation is statistically insignificant. However, the estimation somewhat supports the hybrid version in the mid and lower quantile levels, in which the coefficients for the lagged inflation are significant. Moreover, the results support the view that inflation becomes riskier in the sense of *dispersive order* (Shaked and Shanthikumar (2006)) when increasing expected inflation while decreasing expected inflation makes it less risky. These results indicate that contractionary monetary policy would be more efficient in terms of reducing risks.

The structural break tests detect the existence of a break in all quantile levels, and the break point generally ranges between 1982 and 1983. The pattern of nondecreasing expected inflation coefficient with respect to quantile is still present after the break. As for the lagged inflation coefficient, there is a moderate change after the break: The coefficient is statistically insignificant even in lower quantiles. This result indicates that, when there is a change in the monetary policy, it is expected that the risk changes immediately while there is still a lagged effect around the median.

The remainder of this paper is organized as follows. Section 2 introduces multiple quantile models in the bench mark NKPC. Section 3 shows the results of the empirical analysis and Section 4 concludes.

2. The New Keynesian Phillips Curve and the Conditional Quantile Model

The canonical NKPC model, originated from Calvo (1983)'s staggered price setting together with forward-looking economic agents, expresses current inflation as a function of expected inflation and marginal costs. This model has faced criticism due to insufficient explanation of the persistence of the US inflation dynamics. The hybrid version was introduced to tackle this problem by including backward-looking component (Gali and Gertler (1999)), which can be expressed as

$$\pi_t = \gamma_0 + \gamma_f \pi^e_{t+1} + \gamma_b \pi_{t-1} + \lambda mc_t \tag{2.1}$$

where π_t is the rate of inflation, π^e_{t+1} is expected inflation for $t+1$ at time t, and mc_t is marginal cost of production. If $\gamma_b = 0$, (2.1) is reduced to the canonical NKPC. The choice between the canonical model and the hybrid version has been an important issue not only in theoretic explanation of the inflation dynamics, but also in policy analysis. The presence of lagged inflation in the NKPC indicates the lagged effect of the monetary policy via changing the real economy while the forward-looking term explains the direct effect by changing economic agents' expectation. Consequently, an important policy implication of the hybrid version is that, if the backward-looking component is not significant, the central bank can control current inflation by managing expected inflation with little distortion of the real economy. Gali and Gertler (1999), Gali, Gertler, and López-Salido (2001), and Sbordone (2002, 2005) show that expected inflation is an important driving force of current inflation, while Fuhrer and Moore (1995), Fuhrer (1997), and Roberts (1997) insist that lagged inflation is a more important component than expected inflation in explaining actual inflation.

The main purpose of this paper is to examine the hybrid NKPC at multiple quantile levels. To this end, we introduce the conditional quantile equation to the NKPC. Let $x_t = (1, \pi^e_{t+1}, \pi_{t-1}, mc_t)$ and z_t be the vector of instrumental variables for x_t. In addition, let π_t have the conditional distribution function $F_t(\pi) = Pr(\pi_t \leq \pi | \Omega_t)$, where $\Omega_t = \{\mathfrak{F}_{t-1}, z_t\}$ and $\mathfrak{F}_{t-1}$ is the information set at $t-1$. The αth quantile of π_t conditional on Ω_t, denoted as q^α_t, is defined as

$$\begin{aligned} & q^\alpha_t \equiv \inf_{v \in R} \{v : F_t(v) > \alpha\} \\ & \text{or if } F_t(\pi) \text{ is continuous,} \quad q^\alpha_t \equiv F_t^{-1}(\alpha) \end{aligned} \tag{2.2}$$

That is, conditional quantile q_t^α is that the probability of π_t being less than q_t^α is α. In this paper we assume that there exists a parameter vector $\beta^\alpha = (\gamma_c^\alpha, \gamma_f^\alpha, \gamma_b^\alpha, \lambda^\alpha)'$ for all $0 < \alpha < 1$ such that the NKPC in Equation (2.1) applies to the quantile function q_t^α, i.e. $q_t^\alpha = \gamma_c^\alpha + \gamma_f^\alpha \pi_{t+1}^e + \gamma_b^\alpha \pi_{t-1} + \lambda^\alpha mc_t$. Then the conditional quantile model can be rewritten in a more familiar formulation as

$$\pi_t = x_t' \beta^\alpha + \epsilon_t^\alpha \qquad t = 1, \ldots, T \tag{2.3}$$

where ε_t^α has the following quantile restriction of $Pr(\varepsilon_t^\alpha < 0|\Omega_t) = \alpha$. The data generating process of (2.3) is not truly distinctive to common linear regression models. Indeed, many traditional conditional mean equations can be transformed to (2.3). For example, consider the conventional NKPC model (2.1) with heteroscedastic error.

$$\pi_t \;=\; x_t' \beta + x_t' \gamma \epsilon_t \tag{2.4}$$

where β is the mean parameter, and ϵ_t is iid with $E[\epsilon_t|\Omega_t] = 0$. The conditional quantile of π_t in (2.4) is then simply $q_t^\alpha \;=\; x_t'\beta \;+\; x_t'\gamma \cdot q_t^{\alpha,\epsilon} \;=\; x_t'\beta_t^\alpha$, where $q_t^{\alpha,\epsilon}$ is the αth quantile of ϵ_t and $\beta^\alpha \;=\; \beta + \gamma q_t^{\alpha,\epsilon}$. Thus, the quantile parameter β^α is determined by the mean parameters β, the scale parameters γ, and other parameters that determine the shape of the conditional distribution.

This comparison clarifies a motivation to consider the quantile model in that the conditional mean model is insufficient to make inferences about the risks of inflation. The measurement and management of uncertainty have been an important issue in macroeconomics. For this reason, many of the central banks prefer density forecasts of inflation, such as the inflation fan chart, to point forecasts in the sense that the former contain the uncertainty structure of the forecasts. Conditional quantiles have more information than simply the conditional mean because they include information about the uncertainty structure of the variable of interest such as skewness, kurtosis, and any other factors determining the shape of the distribution.

Another strength to consider multiple quantile model is that doing so captures the asymmetric monetary policy. The conventional mean equation assumes symmetry. For instance, the effects of inflation on the expected inflation gap and the expected deflation gap are equivalent as long as the size of the gap is the same; thus, the Fed's responses to inflationary and deflationary pressures are identical. However, if a model contains the equations for multiple quantiles, it is possible to capture the asymmetry of the responses in two ways: First, the responses of monetary authorities are asymmetric. It is reasonable to consider that the monetary authorities are interested in different quantiles depending on different economic circumstances. For example, suppose a central bank adopts the inflation targeting system to commit it to stabilizing the inflation in a specific range. Then, if the central bank expects inflationary risk in the near future, it must focus more on the probability that the future inflation will exceed the upper bound of the target range. Accordingly, the equations for upper quantiles are more useful in implementing a policy reaction. Consequently, if the coefficients for the NKPC are different between lower and upper quantiles, the Fed's reaction should be different according to whether it is an inflationary or a deflationary pressure periods.

Second, the policy effect on the risk of inflation is asymmetric in that having monotonic quantile coefficients with respect to the level of quantile implies that stochastic orders that measure the risk asymmetrically change with a change in the covariates. A process is considered to be less risky (or less uncertain) if the probability distribution is less dispersed. More precisely, X is defined to be smaller than Y in dispersive order if for all $0 \leq \alpha_1 \leq \alpha_2 \leq 1$, the quantiles for X and Y, denoted as q^{α}_{Xt} and q^{α}_{Yt}, respectively, satisfy $q^{\alpha_2}_{Xt} - q^{\alpha_1}_{Xt} \geq q^{\alpha_2}_{Yt} - q^{\alpha_1}_{Yt}$ and the inequality holds for at least one pair of αs. Suppose π^e_{t+1} is increased by 1% point, then the quantile distances of π_t are changed by $\gamma^{\alpha_2}_f - \gamma^{\alpha_1}_f$. Thus, if $\gamma^{\alpha_i}_f$ is monotonically increasing with α_i, the distances between quantiles get wider with the increase in π^e_{t+1}, which implies that the risk of π_t becomes larger in the sense of dispersive order. That is, an expansionary monetary policy that increases π^e_{t+1} may also increase the uncertainty of inflation while a tightening monetary policy decreases it. On the other hand, if the lower and upper quantile coefficients are different, a change in the value of covariate spreads the tails of distribution in different ways depending on the sign of the change, which alters the risk structure in asymmetric ways.

This asymmetry is also viewed as capturing various types of asymmetric loss function in forecasting. As Granger and Newbold (1986) point out, although an assumption of symmetry about the conditional mean is likely to be an easy one to accept, a symmetry assumption for the loss function is much less acceptable and the corresponding practitioners are more likely to use asymmetric loss functions. In this case, the loss function for forecasting π_{t+i} is generally defined as

$$L(\hat{\epsilon}_{t+i}) = L_1(\hat{\epsilon}_{t+i})1_{\{\hat{\epsilon}_{t+i}>0\}} + L_2(\hat{\epsilon}_{t+i})1_{\{\hat{\epsilon}_{t+i}<0\}} \tag{2.5}$$

where $\hat{\epsilon}_{t+i}$ is the forecast error of π_{t+i}. One popular choice for the asymmetric loss function is the *LinLin* loss function suggested by Granger (1969), in which $L_i = a_i|\hat{\epsilon}_{t+i}|$. If the loss function is *LinLin*, the optimal forecast is equivalent to the forecast of the a_2^{th} conditional quantile. That is, when the linear quantile model is constructed for forecasting purposes, the conditional quantile model is consistent with the optimal linear forecasting in which practitioners put more weight on positive/negative forecast error, that is, for $\alpha < 0.5$, they are concerned with the possibilities that the actual process is less than the forecasted value. It is reasonable to assume that central banks are more concerned about stabilizing inflation and macroeconomy, while governments are more likely to boost the economy by emphasizing on the growth at the cost of modest inflation. Thus, it is expected that central banks will be more interested in the upper quantile of the inflation process, while governments concern more with the lower quantiles.

3. Empirical Analysis

3.1. **Data Description and Issues**

Some empirical issues have arisen in the choice of data in the hybrid NKPC. First, the empirical analysis of the forward-looking behavior is affected by what is used as a proxy for expected inflation. A traditional method is to use realized future inflation data (Gali and Gertler (1999)). However, using actual π_{t+1} may induce a measurement error (Zhang, Osborn, and Kim (2007)), and often causes the problem of weak identification of the instruments (Mavroeidis (2006)). Instead, Zhang, Osborn, and Kim (2009) and Adam and Padula (2011) use observed inflation expectation data which can mitigate the problems. Following the latter, we use the US Survey of Professional Forecast (SPF) of GDP deflator inflation as a proxy for π^e_{t+1}. As introduced by Croushore (1993), SPF is useful for monetary policy analysis and for measuring the response of expectations to policy change.

Another issue is the measure of the marginal cost of firms. Labor income share and output gap are widely used as proxies for real marginal cost. Gali and Gertler (1999) insist that labor income share is a more suitable measure than output gap for real marginal cost, while Neiss and Nelson (2005) argue that output gap better explains the inflation dynamics of the NKPC when it is estimated in theory-consistent manner. We consider both measures to calculate the marginal cost by using real GDP and non-farm unit labor cost (ULC). GDP is detrended using either the CBO's PGDP or Hodrick-Prescott filtering. ULC is detrended using Hodrick-Prescott filtering. Consequently, we estimate NKPC using three different marginal cost data sets. The data used in the estimation span the quarterly period between 1969:I and 2008:II. All data are seasonally adjusted. Detailed data descriptions are provided in Table 6.

Figure 1 shows the GDP deflator inflation and the GDP gap from CBO, with the GDP gap on the left vertical axis and the right vertical axis indicating inflation. As can be seen from the figure, actual inflation has been stable at a low level since the mid-1980s. On the other hand, the GDP gap has expanded its cycle to about 10 years. Clearly, the GDP gap shows a different pattern between the high inflation period, before the mid-1980s, and the low inflation periods, after the mid-1980s, when the second oil shock ended. This result has two possible interpretations. First, the empirical analysis may require structural breaks

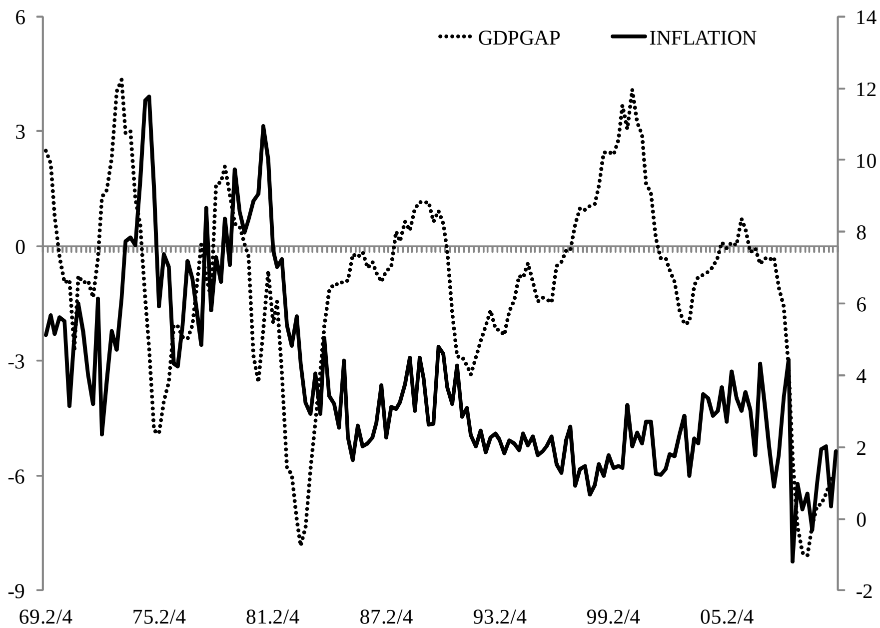


FIGURE 1. **GDP deflator inflation and GDP gap**

because of the change in the economic environments. Second, the traditional conditional mean model cannot explain the economic surroundings because the relationship between inflation and the GDP gap appears differently according to the levels of inflationary pressure. This section empirically examines the possibilities of both interpretations.

Figure 2 represents the GDP deflator inflation and SPF inflation forecasts and shows that the actual inflation and the inflation forecasts have very similar patterns. However, when actual inflation increases, the inflation forecasts are usually less than the actual inflation. When the actual inflation decreases, the inflation forecasts are greater than the actual inflation. That is, the relation between actual inflation and the inflation forecast differs according to the level of inflationary pressure. Thus, multiple quantile analysis can be useful to analyze the various economic situations.

3.2. **Empirical Findings**

This section estimates the US NKPC at multiple quantile levels. Although using SPF mitigates the endogeneity problem, as Zhang, Osborn, and Kim (2009) point out, it may not completely overcome the endogeneity bias because (1) the current output gap is endogenous in that a demand shock causes both GDP and the noise in NKPC, and (2) SPF is possibly correlated with the noise in that one can observe the noise when performing the current

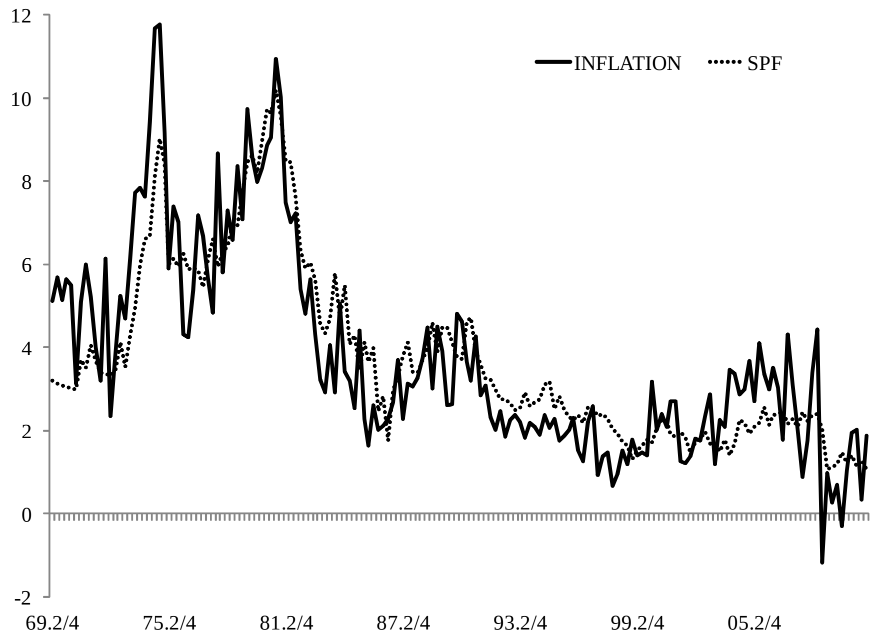


FIGURE 2. **GDP deflator inflation and the SPF inflation forecasts**

forecast. Therefore, we estimate the model using *instrumental variable quantile regression (IVQR)* of Chernozhukov and Hansen (2008) in which the four lags of inflation, interest rate spread (10-year treasury bill rate - 3-month treasury bill rate), the nominal wage growth, and the measure of the marginal cost are used as instruments.

The basic idea of the IVQR method is that, from the quantile condition $Pr[\pi_t < \gamma_c^\alpha + \gamma_b^\alpha \pi_{t-1} + \gamma_f^\alpha \pi_{t+1}^e + \lambda^\alpha y_t | \Omega_t] = \alpha$, if we regress the quantile of $\pi_t - \bar{\gamma}_f^\alpha \pi_{t+1}^e - \bar{\lambda}^\alpha y_t$ on π_{t-1} and z_t, then $\bar{\gamma}_f^\alpha$ and $\bar{\lambda}^\alpha$ will be close to their true values if the coefficients for z_t are close to zero. Consequently, the estimation is done by searching for values of $(\bar{\gamma}_f^\alpha, \bar{\lambda}^\alpha)$ that make the coefficient for z closest to zero. We set the search range for γ_f^α as $[0,\ 1.5]$, and for λ^α as $[-0.1,\ 1.0]$ with an interval of 0.01, respectively. Because the structural equation contains only the first lag of the inflation, it is reasonable to consider a possible serial correlation of the error process as well as the heteroscedasticity. The covariance estimator suggested by Chernozhukov and Hansen (2008) is robust to heteroscedasticity but is not consistent under the existence of the serial correlation. Thus, it is desirable to seek an alternative estimator that is heteroscedasticity and autocorrelation consistent (HAC). In this paper, we apply Fitzenberger (1997)'s *moving blocks bootstrap (MBB)* which is shown to have a HAC property in quantile regression. Because our framework of IVQR is a generalization of the quantile regression, the MBB standard error will obtain the HAC property. We

Table 1. **Estimation results using GDP (full sample)**

α	CBO's PGDP π^e_{t+1}	CBO's PGDP π_{t-1}	CBO's PGDP y_t	CBO's PGDP c	H-P filtered trend π^e_{t+1}	H-P filtered trend π_{t-1}	H-P filtered trend y_t	H-P filtered trend c
0.1	0.530**	0.377**	0.010	-0.788	0.600**	0.328**	0.030	-0.817
	(0.184)	(0.105)	(0.167)	(0.369)	(0.131)	(0.126)	(0.077)	(0.418)
0.2	0.620**	0.339**	0.070	-0.559	0.590**	0.335**	0.100	-0.499
	(0.125)	(0.084)	(0.037)	(0.335)	(0.099)	(0.081)	(0.058)	(0.207)
0.3	0.520**	0.451**	0.100	-0.431	0.560**	0.364**	0.120*	-0.351
	(0.159)	(0.034)	(0.120)	(0.424)	(0.119)	(0.101)	(0.060)	(0.206)
0.4	0.540**	0.458**	0.110	-0.256	0.540**	0.402**	0.180*	-0.151
	(0.201)	(0.156)	(0.120)	(0.607)	(0.131)	(0.116)	(0.086)	(0.235)
0.5	0.500*	0.497**	0.140	0.048	0.480*	0.490**	0.150	0.116
	(0.211)	(0.153)	(0.156)	(0.739)	(0.128)	(0.118)	(0.115)	(0.222)
0.6	0.730**	0.357**	0.230**	0.014	0.700**	0.362*	0.220	-0.022
	(0.136)	(0.117)	(0.071)	(0.227)	(0.146)	(0.138)	(0.117)	(0.253)
0.7	0.810**	0.395**	0.190**	-0.107	0.830**	0.314*	0.210*	0.027
	(0.131)	(0.114)	(0.070)	(0.264)	(0.151)	(0.157)	(0.111)	(0.299)
0.8	1.090**	0.208	0.220*	-0.156	1.220**	0.158	0.230*	-0.057
	(0.118)	(0.168)	(0.074)	(0.347)	(0.107)	(0.209)	(0.117)	(0.576)
0.9	1.370**	0.225	0.220*	0.416	1.220**	0.263	0.020	0.278
	(0.142)	(0.219)	(0.110)	(1.012)	(0.147)	(0.242)	(0.112)	(0.933)
mean eq.	0.694**	0.433**	0.177**	-0.340	0.687**	0.399**	0.178*	-0.255
	(0.089)	(0.075)	(0.028)	(0.157)	(0.092)	(0.082)	(0.056)	(0.147)

Notes: a) Standard errors appear in parentheses.
b) ** and * in π^e_{t+1}, π_{t-1} and y_t columns indicate statistical significance at 1% and 5% levels, respectively.
c) The standard deviation is calculated based on MBB with 500 bootstrap resampling and the block size of 8.
d) The mean equation is estimated by GMM with Newey-West covariance estimator.

use 500 bootstrap resampling with the block size of 8 quarters to estimate the standard deviation of the coefficients.

Note that $IVQR$ is the equation-by-equation single estimation, while our set-up of multiple quantiles is a special form of multiple-equation models. But we maintain the single estimation method because, as Jun and Pinkse (2009) find, the efficiency loss of the single estimation is asymptotically negligible when the regressors are identical for all quantile levels such as ours. Instead, the bootstrap covariance estimator is calculated in the multiple quantile equations set-up, to perform the test of asymmetry across different quantiles.

The estimation results using GDP as a proxy for the marginal cost are shown in Table 1. Substantial differences occur in the effect of the expected inflation across the lower and

Table 2. **Selected test results for differences in π^e_{t+1} coefficients**

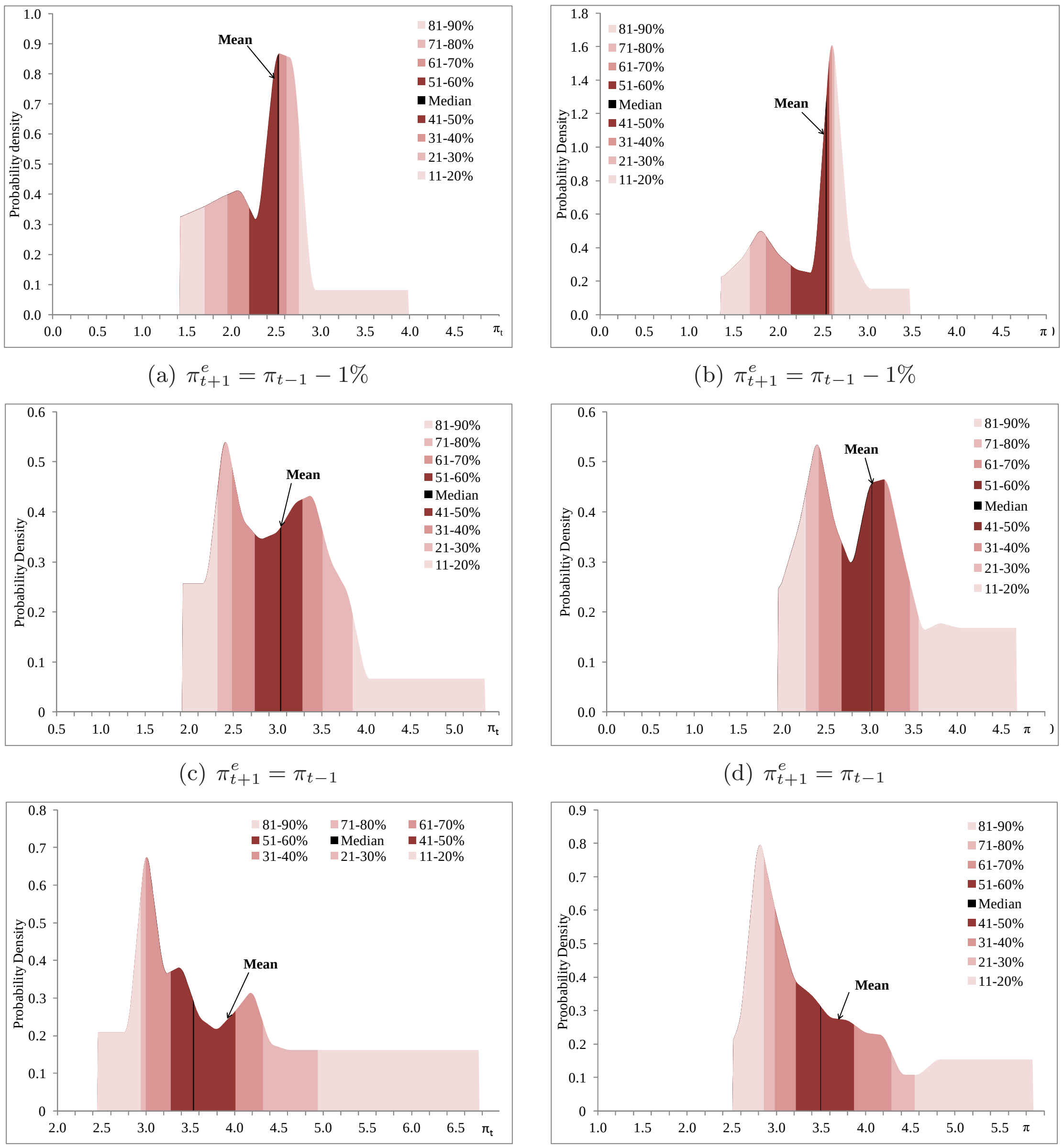
Alt. Hypothesis		$\gamma_f^{0.2} > \gamma_f^{0.1}$	$\gamma_f^{0.6} > \gamma_f^{0.5}$	$\gamma_f^{0.6} > \gamma_f^{0.2}$	$\gamma_f^{0.7} > \gamma_f^{0.5}$
F-stat	PGDP	0.3758	3.0756*	0.3916	3.4321*
	HP filter	0.3900	2.4569	0.3038	4.1241*
Alt. Hypothesis		$\gamma_f^{0.7} > \gamma_f^{0.2}$	$\gamma_f^{0.8} > \gamma_f^{0.5}$	$\gamma_f^{0.8} > \gamma_f^{0.2}$	$\gamma_f^{0.9} > \gamma_f^{0.2}$
F-stat	PGDP	1.1074	6.2689**	6.0025*	42.3483**
	HP filter	1.5488	6.9919**	12.8574**	11.2842**

Notes: a) ** and * indicate statistical significance at 1% and 5% levels, respectively.
b) Tests are done separately for each pair of the coefficients to perform one-sided tests. The tests of joint hypothesis that all $\beta_1^{\alpha_i}$s are equivalent are rejected at 1% level.

upper quantiles: that is, the coefficient for π^e_{t+1} is higher in the upper quantiles compared to the lower quantiles. The coefficient for the lagged inflation is moderately higher in mid-quantiles, but is statistically insignificant in the upper quantiles. The results are similar for all the measures of the marginal cost. Consequently, we find different patterns of the inflation dynamics in lower, mid, and upper quantiles. In the upper quantiles, the inflation is more toward the canonical NKPC in which the backward-looking component is negligible, and the hybrid version fits the mid and lower quantiles better, while the backward-looking component is moderately more important in the mid-quantiles.

As noted in the previous section, the coefficients for π^e_{t+1} and π_{t-1} explain how the Fed's monetary policy affects the inflation. A high coefficient for π^e_{t+1} indicates that the Fed's announcement of a monetary policy change causes the inflation to change faster so that the monetary policy is more effective. Lower coefficients for π_{t-1} indicates that the lagged effect of the monetary policy via the change in the real economy is relatively weaker so that the monetary policy affects inflation with little distortion of the real economy. The estimation result indicates that the Fed's monetary policy concerning inflationary pressure can quickly decrease the upper risks. For instance, many central banks adopting inflation targeting system intend for the inflation to stay within a target range. Thus, the risk that the inflation will be outside the range is its primary concern. Our estimate results indicate that, when the economy is currently facing an inflation, the monetary policy effectively eliminates the risk faster than what we expect based on the point path of the inflation.

As note in Section 2, a monotonic increasing γ_f^α implies that the overall uncertainty of π_t becomes larger as π^e_{t+1} increases in the sense of dispersive order. To examine the monotonic increasing γ_f^α, we also perform the test of $H_1 : \gamma_f^{\alpha_2} - \gamma_f^{\alpha_1} > 0$ for all possible αs of which selected test results are shown in Table 2. We find no evidence of decreasing coefficient with increasing α, and, in most upper α_2 with mid and lower α_1, the tests indicate $\gamma_f^{\alpha_2}$



(a) $\pi^e_{t+1} = \pi_{t-1} - 1\%$ (b) $\pi^e_{t+1} = \pi_{t-1} - 1\%$

(c) $\pi^e_{t+1} = \pi_{t-1}$ (d) $\pi^e_{t+1} = \pi_{t-1}$

(e) $\pi^e_{t+1} = \pi_{t-1} + 1\%$ (f) $\pi^e_{t+1} = \pi_{t-1} + 1\%$

<CBO's GDP GAP > < HP filtered GDP GAP >

note: 1. π_{t-1} is set at 3%.

2. If the quantiles are overlapped, we set the quantile difference at 0.1% point.

FIGURE 3. **Conditional densities given various π^e_{t+1} using GDP**

is greater. Consequently, within the chosen nine quantiles, our test results do not counter to the argument that decreasing expected inflation can reduce the overall uncertainty of inflation, and vice versa.

The dependency of the shape of the conditional distribution with respect to the economic condition is clearer if we look at the conditional density functions at the various levels of expected inflation. We draw rough estimates of the conditional densities by transforming nine quantile levels to a density function. Note that a quantile is, by definition, the inverse mapping of the cumulative distribution function. Therefore, we can obtain the density function via inverse mapping if we have sufficient levels of quantiles. Our transformation is a rough approximation in that we have only nine observations in this inverse mapping: that is, our figure is based on nine observations of the probability distribution function. In charting the densities, we assume that the inflation at $t-1$ is the historical average, 3%, and examine the case when the expected inflation is 1% point above, equivalent to, and 1% point below π_{t-1}, respectively. For simplicity, we disregard the effect of output and focus on the effect of forward-looking component.

Figure 3 shows the conditional densities using the CBO's PGDP, in which the bands with different shades indicate different quantile levels. In each graph, the median is represented with the bold line. The two darkest bands on either side of the median represent equal probability density (10%), resulting in a 20% confidence band around the median. That is, the probability that the inflation lies in this range is 20%. In the same way, adding the next two darkest shades creates a 40% confidence band, and the largest bands indicate an 80% one. The same color bands are not of equal width if the risks are unbalanced. If we regard the median as the central projection, then a wider band on one side indicates that more risks occur to that side in that there is a greater chance that the actual inflation in that direction is far from the central projection. For example, if one is concerned with the 20% confidence band around the median, in Figure 3(a), then one does not have to worry about inflation exceeding the median, but should be concerned with the other direction that is up to 0.3% point below the median.

The figure shows that the shape of the conditional distribution clearly depends on the value of π^e_{t+1}. The density is positively skewed if there is deflationary pressure ($\pi^e_{t+1} < \pi_{t-1}$), and the skewness moves downward as π^e_{t+1} goes up. In addition, there is a substantial change in the upper part of the distribution when π^e_{t+1} shifts, while the downward part is stable. That is, a higher expected inflation than π_{t-1} spreads out the upper tails of the conditional distribution, while a lower one causes little change. This asymmetry in the figure provides useful information about the asymmetric effect of the monetary policy, shifting expected inflation. For example, suppose the Fed reacts to increasing inflationary pressure by announcing a policy change to suppress the inflation. If the announcement affects economics agents' expectation of the future inflation path, the conditional distribution would shift from Figures 3(c) or 3(d) (3(e) or 3(f)) to 3(a) (3(b)), if we disregard the level of π_{t-1} and focus on the change in $\pi^e_{t+1} - \pi_{t-1}$. We can also view 3(e) and 3(f) as the risk effect of the opposite case: easing the monetary policy. The figures show that the upper risks are reduced significantly after the announcement of a tightening monetary policy compared with little change in the lower risk. When comparing 3(c) to 3(a), the 60%-70%, 70%-80% and 80%-90% bands are decreased by 0.08, 0.28, and 0.34 percent point, respectively. On the other hand, the expansionary policy to overcome recession has little effect on the downside risk while increasing the risk of inflation. Consequently, if we assume that the effect of the monetary policy on expected inflation is symmetric, the monetary policy is more effective in eliminating the risks when tightening rather than easing. The asymmetric pattern is unchanged even if we view the conditional mean as the central projection although the effect is milder. Since the asymmetry is due to the nondecreasing expected inflation coefficient, the results is still valid even when the previous inflation is lower so that an expansionary policy is more reasonable. We chart a distribution when $\pi_{t-1} = 2\%$ as shown in Figure 7, which shows that the asymmetric pattern is unchanged.

In the above estimation, we exclude a possible structural break in each quantile NKPC. However, many researchers have detected structural breaks in the inflation processes. Clark and McCracken (2006) find a break in 1982, while Estrella and Fuhrer (2003) suggest another break in 1984. Furthermore, Kim and Kim (2008) find that the backward-looking component is negligible if structural breaks at 1977 and 1982 are explicitly considered in the hybird NKPC. Accordingly we perform the structural break test, estimate the break point and finally modify the NKPC including the detected structural breaks. A few studies propose structural break test methods in quantile equations. Qu (2008) applies *supF* and *expF* type tests to quantile regressions. Lee (2010) suggests the quantile counterpart of Elliott and Müller (2006)'s point optimal test. This paper uses the *supF* test because it provides a break point estimate as a by-product of the test.

Qu (2008)'s *supF* test does not explicitly consider the instrumental variable case. However, it is not difficult to show that the test is valid for instrumental variable estimation if the coefficient estimators in the split samples are asymptotically normal under the null hypothesis of no structural break, which is proved in Chernozhukov and Hansen (2008). Rather than performing a single Bai-Perron type test in all quantiles, we perform the test independently at each quantile level to examine the specific property of the structural breaks across different quantiles.

Table 3 shows the structural break test results. The tests reject no structural break at most quantile levels regardless of the choice of the measure of the marginal cost. In most cases, the test detects a break between 1982 and 1984. The break point is consistent with the existing tests for the conditional mean coefficients. However, we could not find a distinctive pattern in the break points across different quantiles. Note that a change in the

Table 3. **Test results for structural breaks**

	CBO's PGDP		GDP H-P filter		ULC H-P filter	
α	supF	break point	supF	break point	supF	break point
0.2	56.90**	82:II	81.45**	81:II	33.01**	82:IV
0.3	85.72**	82:III	73.51**	83:IV	23.90**	83:IV
0.4	70.94**	82:II	40.76**	83:III	138.51**	81:IV
0.5	161.81**	82:III	63.15**	83:IV	65.09**	82:I
0.6	157.60**	84:III	54.02**	83:IV	51.57**	82:I
0.7	42.52**	83:II	74.02**	82:II	78.26**	82:I
0.8	34.14**	82:III	49.60**	82:III	64.05**	83:I

Notes: ** and * indicates that the test rejects the hypothesis of no structural break at 1% and 5% levels, respectively.

conditional mean indicates a shift in distribution function, thereby a shift in all quantiles. Accordingly, it is natural that we observe a structural break in all quantiles if there is a structural break in the conditional mean.

To examine the effect of the structural break across quantiles, we estimate the model using the data from 1984:I. Roughly, the pre-break period was dominated by high inflation while stable inflation was common after the break. Table 4 shows the result using the GDP in the post-break period. The overall pattern of $\hat{\gamma}_f^\alpha$ is similar to the full sample case, although the monotonic nondecreasingness is less clear. The coefficient for π_{t+1}^e is around 0.5 when q is less than 0.8 while it is substantially higher in the upper quantiles. Consequently, the downside risk is stable with respect to the change in π_{t+1}^e, but there is a substantial asymmetric change in the upside risk in response to a tightening or an expansionary monetary policy. As for the π_{t-1} coefficient, there is a considerable variation across quantiles: The coefficient is greater and statistically significant around the median, supporting the hybrid version of NKPC, and is close to zero and insignificant when the quantile goes further toward the end of the distribution. These results indicate that, when a change in the monetary policy occurs, it is expected that the risk responds more quickly while there is a lagged effect around the median. The median estimation results coincide with the conditional mean estimation results in that the lagged effect is statistically significant, but this finding is counter to that of Kim and Kim (2008).

Figure 4 shows the conditional densities for post-break inflation when the GDP gap is used as a proxy for the marginal cost. There are moderate changes after focusing on post-break data: We find a symmetry or a mild asymmetry in the risk structure when the confidence bands are 20% and 40%. However, a substantial asymmetry still exists in the larger confidence bands.

Table 4. **Estimation results using GDP (1984:I -)**

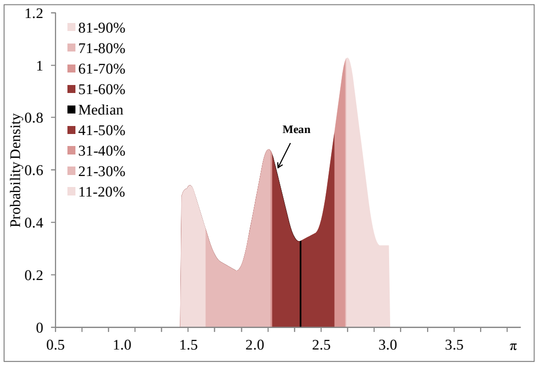
	CBO's PGDP				H-P filtered trend			
α	π^e_{t+1}	π_{t-1}	y_t	c	π^e_{t+1}	π_{t-1}	y_t	c
0.1	0.405**	0.186	0.060	-0.065	0.360*	0.160	0.050	0.302
	(0.129)	(0.117)	(0.060)	(0.342)	(0.153)	(0.115)	(0.090)	(0.419)
0.2	0.435**	0.178	0.020	0.225	0.540**	0.035	0.100	0.254
	(0.104)	(0.124)	(0.044)	(0.315)	(0.096)	(0.106)	(0.075)	(0.263)
0.3	0.390**	0.301**	0.040	0.439	0.490**	0.148	0.080	0.354
	(0.100)	(0.118)	(0.039)	(0.300)	(0.093)	(0.117)	(0.070)	(0.284)
0.4	0.465**	0.197	0.040	0.476	0.430**	0.241*	0.100	0.432
	(0.103)	(0.121)	(0.048)	(0.277)	(0.092)	(0.110)	(0.081)	(0.256)
0.5	0.420**	0.357**	0.020	0.426	0.350**	0.489**	0.080	0.559
	(0.078)	(0.099)	(0.062)	(0.254)	(0.088)	(0.101)	(0.088)	(0.276)
0.6	0.360**	0.413**	0.040	0.641	0.350**	0.477**	0.060	0.469
	(0.076)	(0.071)	(0.056)	(0.268)	(0.094)	(0.081)	(0.087)	(0.312)
0.7	0.450**	0.414**	0.040	0.548	0.830**	0.363*	0.040	-0.231
	(0.128)	(0.091)	(0.060)	(0.342)	(0.156)	(0.119)	(0.093)	(0.353)
0.8	0.990**	0.147	0.100	0.295	1.140**	0.060	0.230*	0.014
	(0.243)	(0.161)	(0.090)	(0.519)	(0.236)	(0.190)	(0.106)	(0.547)
0.9	1.060**	0.090	0.120	1.182	1.180**	-0.127	0.220*	0.831
	(0.208)	(0.218)	(0.109)	(0.959)	(0.209)	(0.245)	(0.108)	(0.953)
mean eq.	0.604**	0.203**	0.049**	0.338	0.618**	0.210**	0.074	0.268
	(0.086)	(0.063)	(0.028)	(0.209)	(0.069)	(0.057)	(0.047)	(0.203)

Notes: a) Standard errors appear in parentheses.
b) ** and * in π^e_{t+1}, π_{t-1} and y_t columns indicate statistical significance at 1% and 5% levels, respectively.
c) The standard deviation is calculated based on MBB with 500 bootstrap resampling and the block size of 8.
d) The mean equation is estimated by GMM with Newey-West covariance estimator.

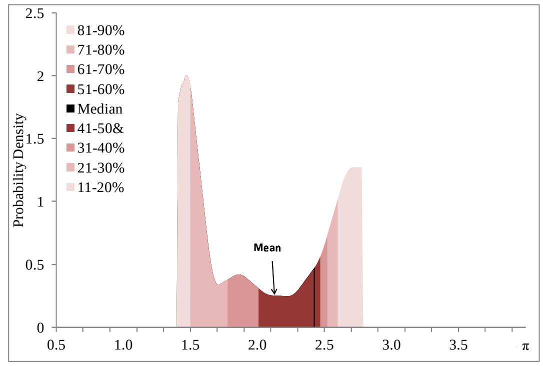
Table 5. **Selected test results for differences in π^e_{t+1} coefficients(1984-)**

Alt. Hypothesis		$\gamma_f^{0.2} > \gamma_f^{0.3}$	$\gamma_f^{0.2} > \gamma_f^{0.5}$	$\gamma_f^{0.2} > \gamma_f^{0.6}$	$\gamma_f^{0.3} > \gamma_f^{0.5}$
F-stat	PGDP	0.0092	0.0911	0.0074	0.2963
	HP filter	0.1515	1.1246	1.3622	0.8099
Alt. Hypothesis		$\gamma_f^{0.3} > \gamma_f^{0.6}$	$\gamma_f^{0.7} > \gamma_f^{0.6}$	$\gamma_f^{0.8} > \gamma_f^{0.2}$	$\gamma_f^{0.9} > \gamma_f^{0.2}$
F-stat	PGDP	0.0094	1.1671	2.8927*	3.5687*
	HP filter	1.5934	7.2000**	3.0769	4.0039*

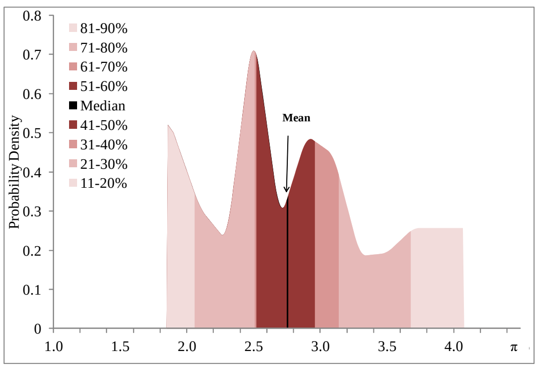
Notes: a) ** and * indicate statistical significance at 1% and 5% levels, respectively.



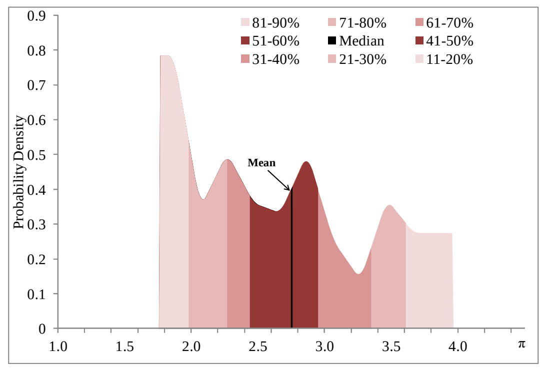
(a) $\pi^e_{t+1} = \pi_{t-1} - 1\%$



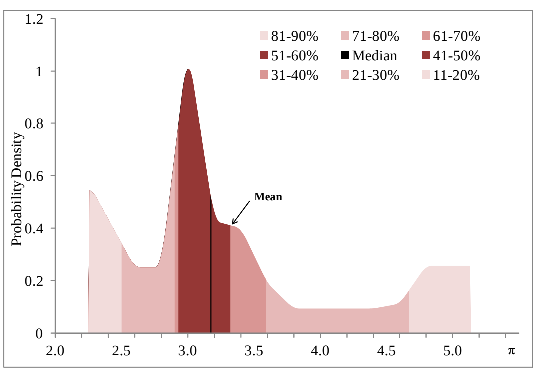
(b) $\pi^e_{t+1} = \pi_{t-1} - 1\%$



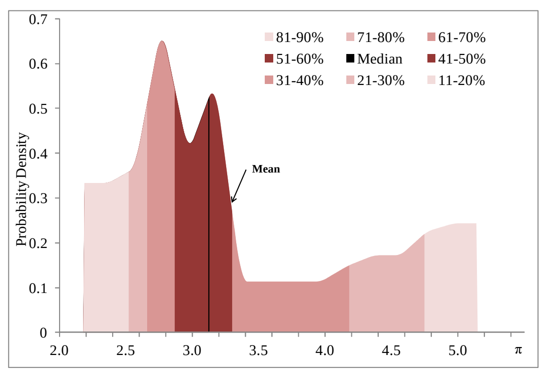
(c) $\pi^e_{t+1} = \pi_{t-1}$



(d) $\pi^e_{t+1} = \pi_{t-1}$



(e) $\pi^e_{t+1} = \pi_{t-1} + 1\%$



(f) $\pi^e_{t+1} = \pi_{t-1} + 1\%$

<CBO's GDP GAP > < HP filtered GDP GAP >

note: 1. π_{t-1} is set at 3%.

2. If the quantiles are overlapped, we set the quantile difference at 0.1% point.

FIGURE 4. **Conditional densities given various π^e_{t+1} using GDP (1984:I -)**

4. Conclusion

In this paper, we examine the hybrid NKPC in multiple quantiles. By using the quantile analysis, we find two important features not captured by the traditional conditional mean analysis. First, the response of inflation with respect to the change in expected inflation is asymmetric across quantiles. The lower quantiles are relatively stable for a change in π^e_{t+1} while the upper quantiles are more sensitive to change. An important implication is that a positive shock on π^e_{t+1} increases the risk of inflation in the sense that it spreads out the distribution, while a negative shock decreases the risk. This asymmetry provides an interesting policy implication that tightening monetary policy is more effective in stabilizing the economy in that it reduces uncertainty.

Second, the role of the backward-looking component strongly depends on the level of the quantiles. After considering the structural break at about 1983, we find that the coefficient of the lagged inflation is significant only in the center of the distribution. Because one implication of the existence of the backward-looking component is the indirect and more complicated effect of the monetary policy by affecting the real economy, the result indicates the possibility that the risk structure will shift against a monetary policy shock more quickly while the point path of the inflation shows a relatively gradual change.

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Appendix A. Tables and Figures

Table 6. Data Description

Variables	quarterly average basis[a)]
π_t	Annualized growth rate of GDP deflator
π^e_{t+1}	The quarterly median forecast of the annualized percent change of GDP implicit price deflator(one-quarter ahead forecast) from SPF
mc_t	1) Percentage deviation of real GDP from the CBO's PGDP or its Hodrick-Prescott (HP) trend y_t 2) Percentage deviation of Non-farm business sector Unit Labor Costs from the Hodrick-Prescott (HP) trend
Instrumental variables	Constant Four lags of π_t, and mc_t 1) and 2) Four lags of the yield spread between long-term and short-term government bond[b)] Four lags of the wage inflation[c)]

Notes: a) All data are obtained from web-based database (FRED) in Federal Reserve Bank of St. Louis.

b) We use yield spread between 10 year and 3 month U.S. Treasury bonds.

c) We use the growth rate of non-farm business compensation

TABLE 7. **Estimation results using ULC (full sample)**

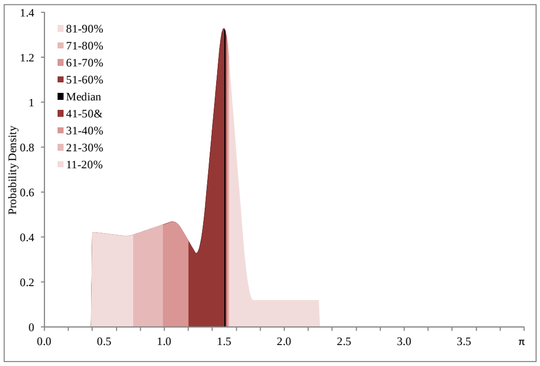
α	π^e_{t+1}	π_{t-1}	y_t	c
0.1	0.700**	0.250	0.140	-0.724
	(0.160)	(0.129)	(0.100)	(0.354)
0.2	0.520**	0.341*	0.120	-0.394
	(0.171)	(0.125)	(0.136)	(0.310)
0.3	0.430*	0.480**	0.120	-0.232
	(0.181)	(0.151)	(0.142)	(0.257)
0.4	0.460**	0.486**	0.220*	-0.063
	(0.190)	(0.163)	(0.127)	(0.229)
0.5	0.380*	0.566**	0.150	0.152
	(0.180)	(0.159)	(0.131)	(0.200)
0.6	0.360**	0.602**	0.150	0.101
	(0.209)	(0.191)	(0.123)	(0.278)
0.7	0.600**	0.557**	0.150	-0.071
	(0.225)	(0.218)	(0.128)	(0.317)
0.8	1.070**	0.195	0.220	0.122
	(0.144)	(0.180)	(0.140)	(0.337)
0.9	1.110**	0.165	0.380*	0.615
	(0.164)	(0.261)	(0.148)	(0.523)
mean eq.	0.605**	0.443**	0.250*	-0.158
	(0.093)	(0.076)	(0.108)	(0.150)

Notes: a) Standard errors appear in parentheses.
b) ** and * in π^e_{t+1}, π_{t-1} and y_t columns indicate statistical significance at 1% and 5% levels, respectively.
c) The standard deviation is calculated based on MBB with 500 bootstrap resampling and the block size of 8.
d) The mean equation is estimated by GMM with Newey-West covariance estimator.

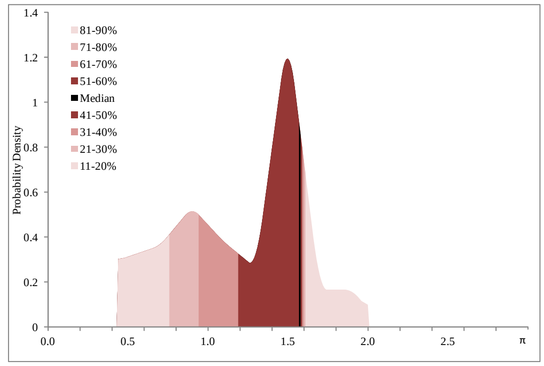
TABLE 8. **Estimation results using ULC (1984:I -)**

α	π^e_{t+1}	π_{t-1}	y_t	c
0.1	0.720**	-0.051	0.240*	-0.266
	(0.140)	(0.121)	(0.106)	(0.440)
0.2	0.510**	0.088	0.180	0.247
	(0.122)	(0.129)	(0.101)	(0.237)
0.3	0.370*	0.210	0.180	0.514
	(0.095)	(0.124)	(0.103)	(0.209)
0.4	0.420	0.294*	0.260**	0.378
	(0.089)	(0.130)	(0.097)	(0.244)
0.5	0.440**	0.426**	0.180*	0.280
	(0.093)	(0.113)	(0.088)	(0.243)
0.6	0.370**	0.460**	0.180*	0.445
	(0.085)	(0.094)	(0.074)	(0.232)
0.7	0.330**	0.512**	0.160	0.518
	(0.139)	(0.104)	(0.085)	(0.306)
0.8	0.990**	0.181	0.160	0.196
	(0.220)	(0.159)	(0.108)	(0.457)
0.9	0.850**	0.029	-0.068	1.385
	(0.217)	(0.219)	(0.120)	(0.987)
mean eq.	0.689**	0.190**	0.266**	0.053
	(0.077)	(0.071)	(0.061)	(0.200)

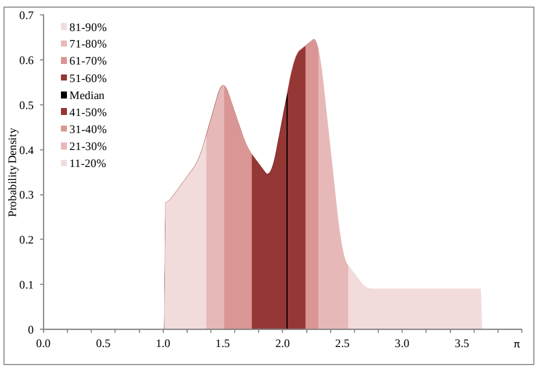
Notes: a) Standard errors appear in parentheses.
b) ** and * in π^e_{t+1}, π_{t-1} and y_t columns indicate statistical significance at 1% and 5% levels, respectively.
c) The standard deviation is calculated based on MBB with 500 bootstrap resampling and the block size of 8.
d) The mean equation is estimated by GMM with Newey-West covariance estimator.



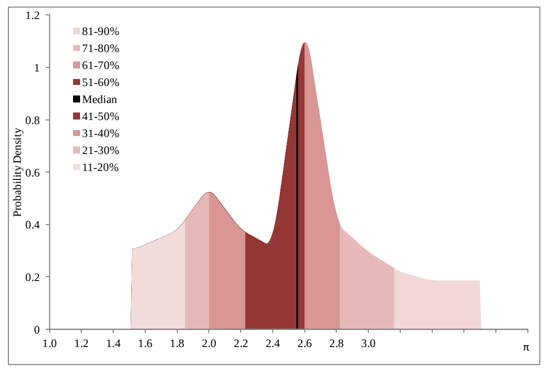
(a) $\pi^e_{t+1} = \pi_{t-1} - 1\%$



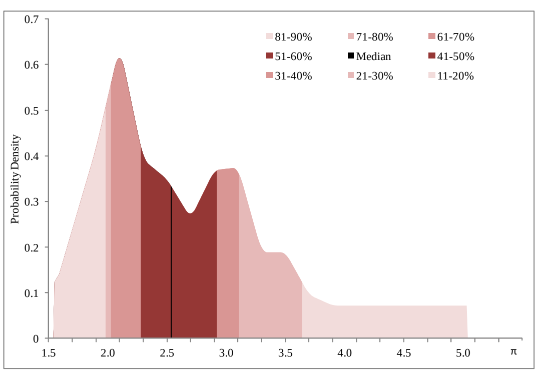
(b) $\pi^e_{t+1} = \pi_{t-1} - 1\%$



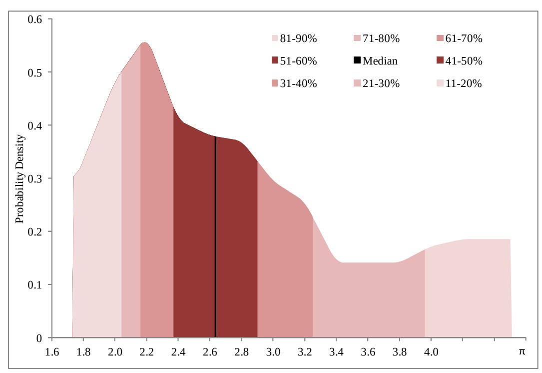
(c) $\pi^e_{t+1} = \pi_{t-1}$



(d) $\pi^e_{t+1} = \pi_{t-1}$



(e) $\pi^e_{t+1} = \pi_{t-1} + 1\%$



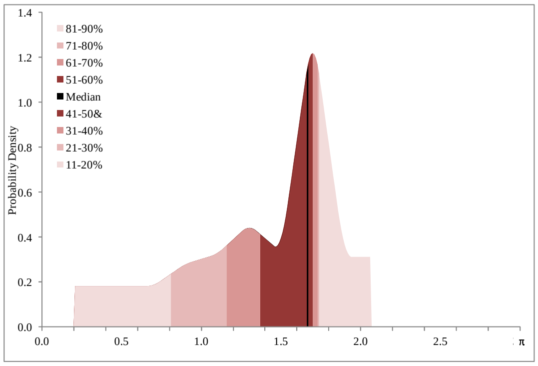
(f) $\pi^e_{t+1} = \pi_{t-1} + 1\%$

<CBO's GDP GAP > < HP filtered GDP GAP >

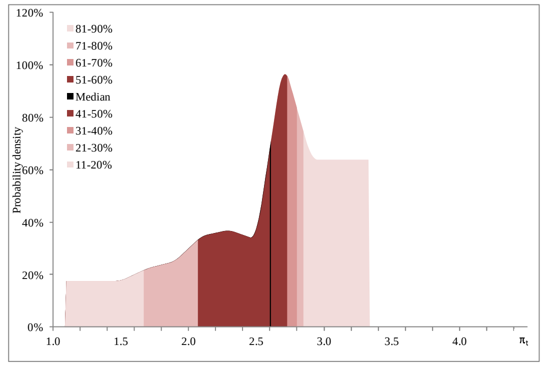
note: 1. π_{t-1} is set at 2%.

2. If the quantiles are overlapped, we set the quantile difference at 0.1% point.

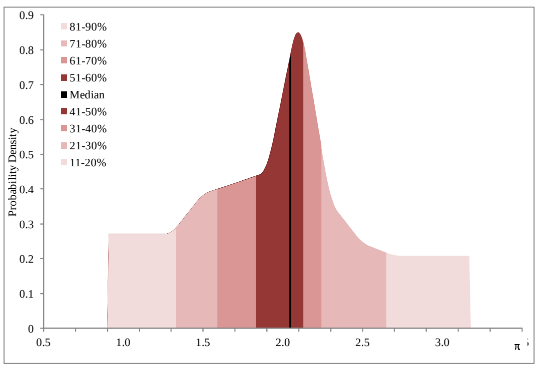
FIGURE 5. **Conditional densities given various π^e_{t+1} using GDP (2)**



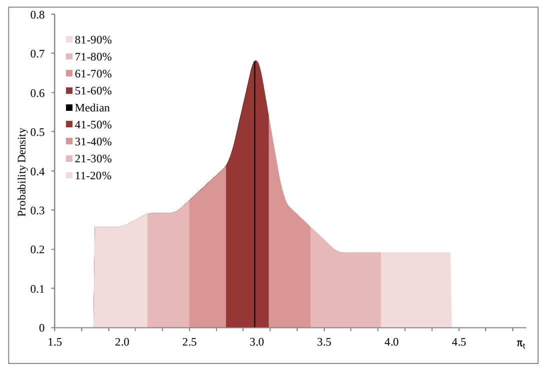
(a) $\pi^e_{t+1} = \pi_{t-1} - 1\%$



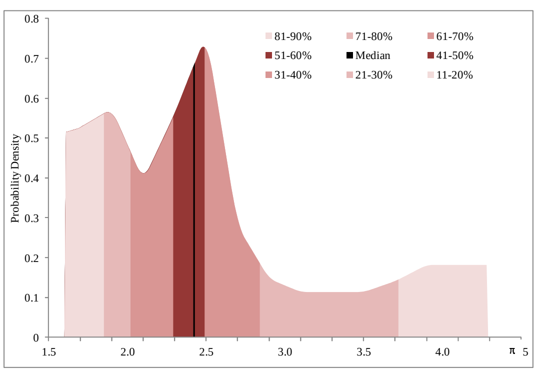
(b) $\pi^e_{t+1} = \pi_{t-1} - 1\%$



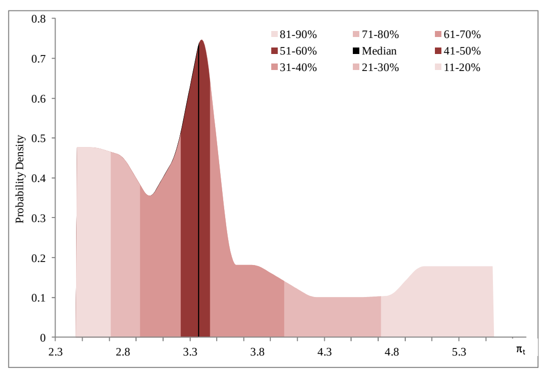
(c) $\pi^e_{t+1} = \pi_{t-1}$



(d) $\pi^e_{t+1} = \pi_{t-1}$



(e) $\pi^e_{t+1} = \pi_{t-1} + 1\%$



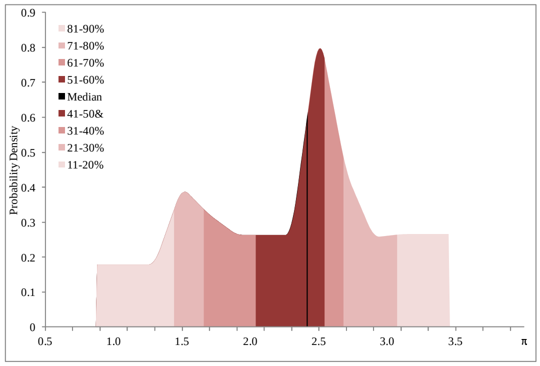
(f) $\pi^e_{t+1} = \pi_{t-1} + 1\%$

$< \pi_{t-1} = 2\% >$ $< \pi_{t-1} = 3\% >$

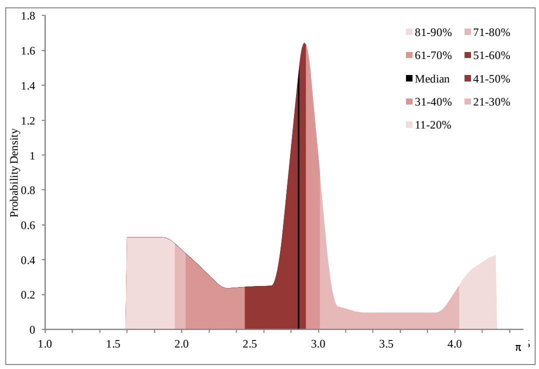
note: 1. π_{t-1} is set at 2%.

2. If the quantiles are overlapped, we set the quantile difference at 0.1% point.

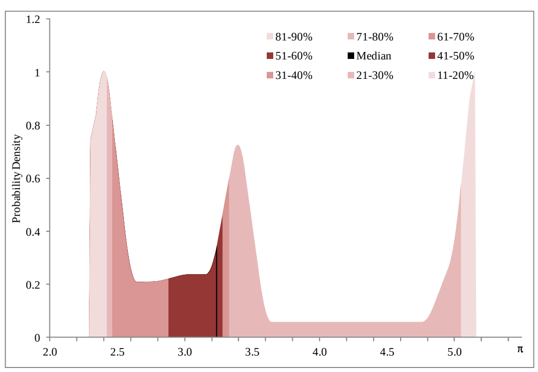
FIGURE 6. **Conditional densities given various π^e_{t+1} using ULC**



(a) $\pi^e_{t+1} = \pi_{t-1} - 1\%$



(b) $\pi^e_{t+1} = \pi_{t-1}$



(c) $\pi^e_{t+1} = \pi_{t-1} + 1\%$

note: 1. π_{t-1} is set at 3%.

2. If the quantiles are overlapped, we set the quantile difference at 0.1% point.

FIGURE 7. **Conditional densities using ULC (1984:I -)**