Housing Dynamics: An International Perspective

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Objectives

(1) Document investment dynamics across countries

- Large cross-country differences
  - U.S. facts exception rather than rule
  - None of the countries consistent with predictions of standard models

(2) Use international facts to guide development of theory

- Model with cross-country differences in mortgage markets
  - Use the model to investigate their quantitative importance
  - First results suggest they are important

Motivation

- Residential investment *leads* GDP in U.S. business cycle
  - Major contributor to slowdown in GDP growth a year before recession
    ▸ Leamer, 2007
  - Most highly correlated with future GDP, 1-2 quarters
- Nonresidential investment *lags* GDP, 2-3 quarters
- Models with disaggregated investment predict the opposite

  e.g., Gomme and Rupert, *JME* 2007; Davis and Heathcote, *IER* 2005; Gomme, Kydland, and Rupert, *JPE* 2001 (various home production models)

Motivation & related literature

- U.S. facts often used in development of theory

(1) Total investment: procyclical & three times as volatile as GDP

(2) Residential twice as volatile as nonresidential

(3) Residential leads while nonresidential lags GDP

Motivation & related literature

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Easy

(2) Residential twice as volatile as nonresidential

Harder

Davis and Heathcote, *IER* 2005 — multisector model

(3) Residential leads while nonresidential lags GDP

Difficult

GKR, *JPE* 2001 — differences in time-to-build (partial success)
Fisher, *JPE* 2007 — GKR w/ complementarities in production

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Fisher, *JPE* 2007 — GKR w/ complementarities in production

- Can these facts/theories be generalized to other countries?

Features of mortgage markets considered

- Fixed rate (FRM) vs. adjustable rate (ARM)
- Duration of mortgage contracts
- Loan-to-value ratios
- Mortgage rate dynamics over the business cycle

Outline of the talk

(1) International facts

(2) Model, calibration, and quantitative findings

International facts

- Countries in our sample:

AUS	59.Q3-06.Q4
BEL	80.Q1-06.Q4
CAN	61.Q1-06.Q4
FRA	71.Q1-06.Q4
UK	65.Q1-06.Q4
US	58.Q1-06.Q4

- Deviations of logged data from HP trend
- Business cycle tables
- Tests of statistical significance

International facts—total investment (GFCF)

Name	Rel. s.d.	Correlations of real GDP in t with a variable in $t+j$: j = -4	-3	-2	-1	0	1	2	3	4
AUS										
BEL										
CAN										
FRA										
UK										
US										

International facts—total investment (GFCF)

Name	Rel. s.d.	Correlations of real GDP in t with a variable in $t+j$: j = -4	-3	-2	-1	0	1	2	3	4
AUS	3.98									
BEL	3.93									
CAN	3.32									
FRA	2.65									
UK	2.55									
US	3.23									

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Name	Rel. s.d.	Correlations of real GDP in t with a variable in $t+j$: j = -4	-3	-2	-1	0	1	2	3	4
AUS	3.98					**0.68**				
BEL	3.93					**0.65**				
CAN	3.32					**0.62**				
FRA	2.65					**0.84**				
UK	2.55					**0.65**				
US	3.23					**0.90**				

International facts—total investment (GFCF)

Name	Rel. s.d.	Correlations of real GDP in t with a variable in $t+j$: j = -4	-3	-2	-1	0	1	2	3	4
AUS	3.98	0.11	0.27	0.44	0.57	**0.68**	0.57	0.43	0.27	0.11
BEL	3.93	0.40	0.45	0.53	0.61	**0.65**	0.64	0.58	0.46	0.31
CAN	3.32	-0.07	0.15	0.38	0.55	**0.62**	0.57	0.45	0.36	0.27
FRA	2.65	0.32	0.50	0.67	0.79	**0.84**	0.79	0.67	0.52	0.37
UK	2.55	0.21	0.33	0.42	0.53	**0.65**	0.62	0.55	0.48	0.35
US	3.23	0.30	0.48	0.68	0.83	**0.90**	0.81	0.65	0.44	0.21

International facts—residential vs nonresidential

Country	Relative std. dev.	Correlations of real GDP in t with a variable in $t+j$: j = -4	-3	-2	-1	0	1	2	3	4
(a): Residential										
AUS										
BEL										
CAN										
FRA										
UK										
US										
(b): Nonresidential: structures (S), equipment (E)										
AUS	S									
BEL	S+E									
CAN	S									
FRA	S+E									
UK	S									
US	S									

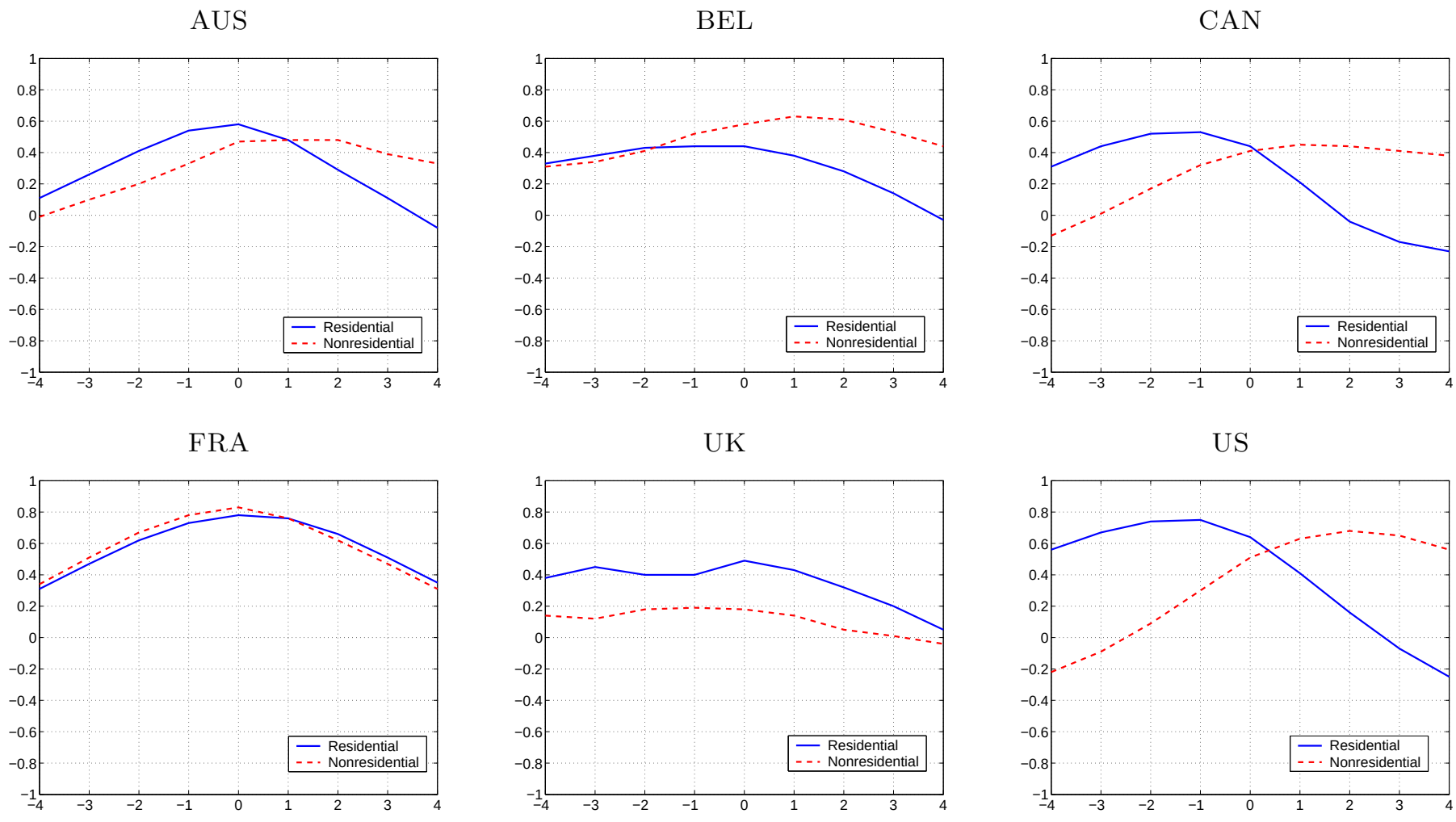
International facts—residential vs nonresidential

Country		Relative std. dev.	Correlations of real GDP in t with a variable in $t+j$: j = -4	-3	-2	-1	0	1	2	3	4
(a): Residential											
AUS		5.95									
BEL		7.97									
CAN		4.39									
FRA		3.05									
UK		5.02									
US		6.42									
(b): Nonresidential: structures (S), equipment (E)											
AUS	S	6.96									
BEL	S+E	4.36									
CAN	S	3.97									
FRA	S+E	3.24									
UK	S	3.24									
US	S	3.40									

International facts—residential vs nonresidential

Country		Relative std. dev.	Correlations of real GDP in t with a variable in $t+j$: j = -4	-3	-2	-1	0	1	2	3	4
(a): Residential											
AUS		5.95	0.11	0.26	0.41	0.54	**0.58**	0.48	0.29	0.11	-0.08
BEL		7.97	0.33	0.38	0.43	0.44	**0.44**	0.38	0.28	0.14	-0.03
CAN		4.39	0.31	0.44	0.52	0.53	**0.44**	0.21	-0.04	-0.17	-0.23
FRA		3.05	0.31	0.47	0.62	0.73	**0.78**	0.76	0.66	0.51	0.35
UK		5.02	0.38	0.45	0.40	0.40	**0.49**	0.43	0.32	0.20	0.05
US		6.42	0.56	0.67	0.74	0.75	**0.64**	0.41	0.16	-0.07	-0.25
(b): Nonresidential: structures (S), equipment (E)											
AUS	S	6.96	-0.01	0.10	0.20	0.33	**0.47**	0.48	0.48	0.39	0.33
BEL	S+E	4.36	0.31	0.34	0.41	0.52	**0.58**	0.63	0.61	0.53	0.44
CAN	S	3.97	-0.13	0.01	0.17	0.32	**0.41**	0.45	0.44	0.41	0.38
FRA	S+E	3.24	0.34	0.51	0.67	0.78	**0.83**	0.76	0.62	0.47	0.31
UK	S	3.24	0.14	0.12	0.18	0.19	**0.18**	0.14	0.05	0.01	-0.04
US	S	3.40	-0.22	-0.09	0.09	0.30	**0.51**	0.63	0.68	0.65	0.56

International facts—residential vs nonresidential



Residential—statistical test of leading vs lagging

Based on bootstrap methods

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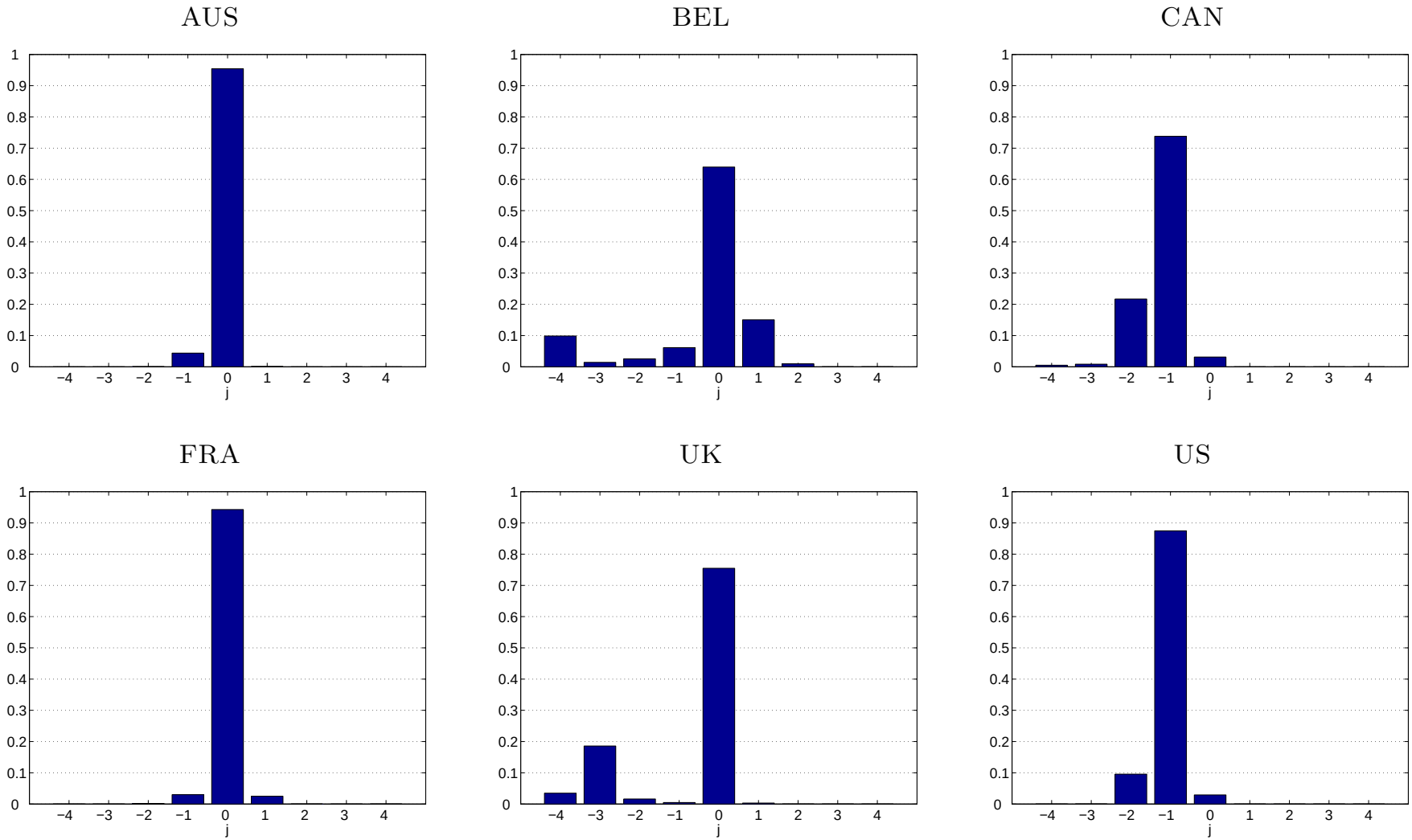
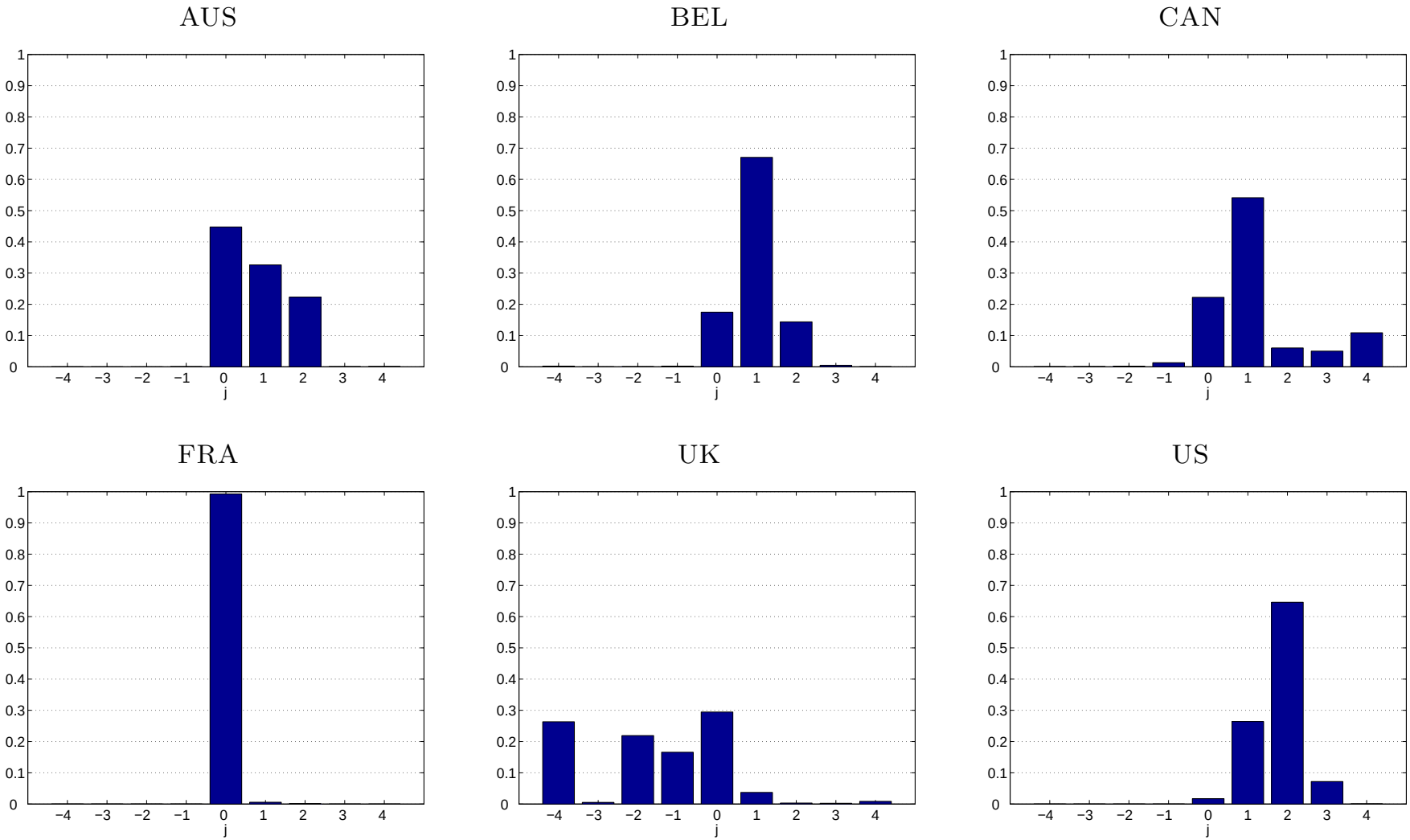
Based on bootstrap methods



The bars represent the frequency of a given cross-correlation being the highest in the cross-correlogram for a bootstrapped data sample.

Nonresidential—stat. test of leading vs lagging

Based on bootstrap methods



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Our terminology

- Leading vs lagging
  - 'Leading'—the highest correlation is at a lead in at least 95% of the bootstrapped samples
  - 'Lagging'—the highest correlation is at a lag in at least 95% of the bootstrapped samples
- FRM vs ARM (standard classification)
  - 'FRM'—interest rate is fixed for at least 5 yrs
  - 'ARM'—interest rate is reset after 1 yr or sooner, or is tied to market rates

Cross-country differences

	Residential leading?	Nonresid. lagging?	Mortg. type	Years for which fixed	Duration (years)	Mortgage debt to GDP ratio (2004)
AUS	×	×	ARM	N/A	25	0.74
BEL	×	×	FRM	10	20	0.31
CAN	✓	×	FRM	5	25	0.43
FRA	×	×	FRM	15	15-20	0.26
UK	×	×	ARM	N/A	25	0.75
US	✓	✓	FRM	30	30	0.69

... may differ also in the dynamics of mortgage rates (will come back to that)

Cross-country differences

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- None of the ARM countries exhibits the US lead-lag
- Not all FRM countries exhibit the US lead-lag
  - Different lengths for which fixed, durations, debt-to-GDP ratios

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US	✓	✓	FRM	30	30	0.69

- None of the ARM countries exhibits the US lead-lag
- Not all FRM countries exhibit the US lead-lag
  - Different lengths for which fixed, durations, debt-to-GDP ratios
- Lead-lag in standard models not supported by any country

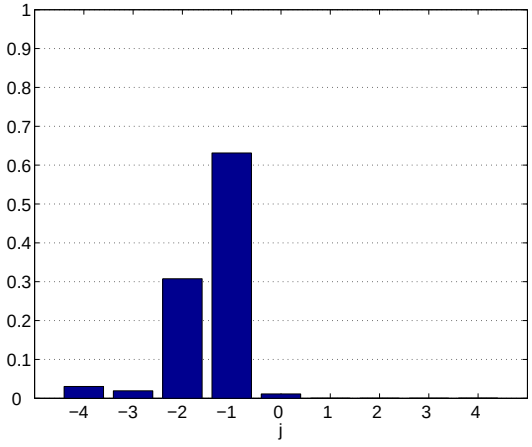
US data—further details

United States

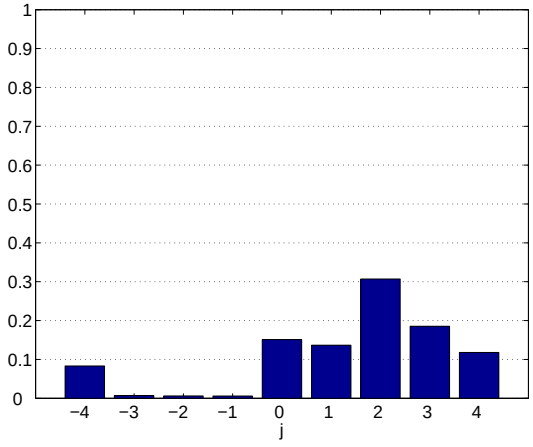
	Relative std. dev.	Correlations of real GDP in t with a variable in $t+j$: j = -4	-3	-2	-1	0	1	2	3	4
RESIDENTIAL INVESTMENT										
1958.Q1–2006.Q4										
Single family	8.77	0.62	0.71	0.76	0.73	**0.60**	0.35	0.08	-0.17	-0.33
Multifamily	11.22	0.16	0.28	0.39	0.47	**0.49**	0.43	0.32	0.19	0.07
S1: 1958.Q1–1983.Q4										
Single family	8.84	0.58	0.65	0.73	0.72	**0.62**	0.39	0.14	-0.11	-0.30
Multifamily	11.40	0.13	0.25	0.38	0.48	**0.51**	0.46	0.34	0.21	0.07
S2: 1984.Q1–2006.Q4										
Single family	8.40	0.51	0.57	0.60	0.57	**0.48**	0.28	0.05	-0.13	-0.25
Multifamily	10.42	-0.02	-0.01	0.07	0.14	**0.22**	0.27	0.31	0.32	0.30
DURABLE GOODS										
Motor vehicles	4.78	0.46	0.52	0.61	0.65	**0.63**	0.40	0.21	-0.01	-0.21
Furniture	2.07	0.34	0.52	0.69	0.82	**0.84**	0.75	0.60	0.45	0.28
Other	2.25	0.25	0.40	0.55	0.69	**0.76**	0.71	0.60	0.44	0.26
NONDURABLES	0.71	0.32	0.49	0.66	0.76	**0.80**	0.70	0.53	0.33	0.11

Statistical test of leading vs lagging, United States

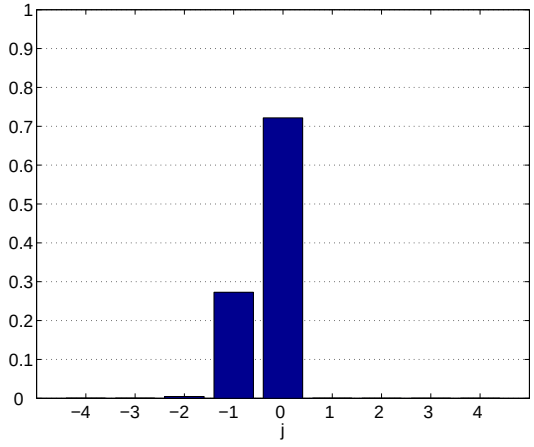
Single family (post-84)



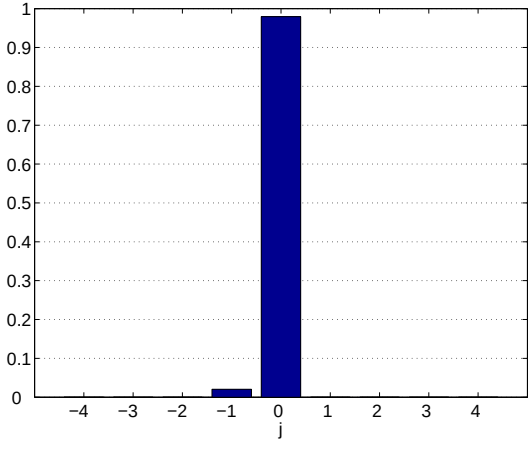
Multifamily (post-84)



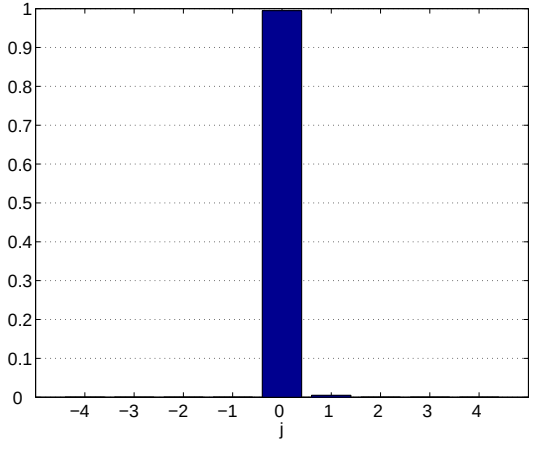
Motor vehicles



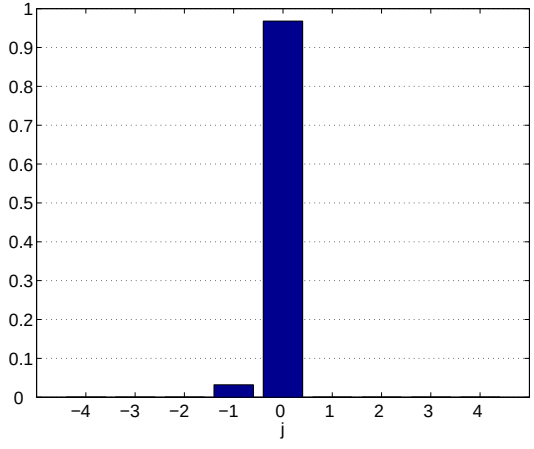
Furniture



Other



Nondurables



- Similar results for Canada

Model—overview

- Purpose: To investigate the quantitative role of mortgages in the cyclical dynamics of residential and nonresidential investment
- Based on market/home production model of GKR, *JPE* 2001
- Exogenous elements
  - **Mortgage rates**—estimated joint process with TFP and inflation rate
  - **Demand for mortgages**—a fraction of residential investment is required to be financed by a mortgage
  - **Form of the mortgage contract**—FRM or ARM

Model—overview (cont'd)

- Tractable model of mortgage contracts
- Equilibrium effects of mortgages: time-varying 'wedge'
- Wedge is a function of
  - Loan-to-value ratio
  - Duration of the mortgage contract
  - FRM vs ARM
  - Expectations of future TFP, inflation, and the mortgage rate

Model—preferences and technology

- Preferences of a representative household

$$V(\mu_t) = u\left[c\left(c_{Mt}, c_{Ht}\right), 1 - h_{Mt} - h_{Ht}\right] + \beta E_t V(\mu_{t+1})$$

- Technology (home and market)

$$c_{Ht} = G(k_{Ht}, h_{Ht})$$

$$c_{Mt} + x_{Mt} + q(x_{Ht})x_{Ht} = \exp(z_{Mt})F(k_{Mt}, h_{Mt})$$

$$x_{Mt} = \sum_{j=1}^{J} \phi_j s_{jt}$$

$$s_{j-1,t+1} = s_{jt} \qquad j = 2, ..., J$$

$$k_{M,t+1} = (1 - \delta_M)k_{Mt} + s_{1t}$$

$$k_{H,t+1} = (1 - \delta_H)k_{Ht} + x_{Ht}$$

Model—mortgages

- Financing and budget constraints

$$I_t = \theta p_t x_{Ht} \qquad \theta \in [0,1]$$

$$c_{Mt} + x_{Mt} + q x_{Ht} = (1-\tau_r) r_t k_{Mt} + (1-\tau_w) w_t h_{Mt} + I_t/p_t - m_t/p_t + \tau_t$$

- Mortgage payments

$$m_t = (R_t + \delta_{Dt}) d_t \qquad \delta_{Dt} \in (0,1) \Rightarrow m_t > R_t d_t$$

$$d_{t+1} = (1-\delta_{Dt}) d_t + I_t$$

$$\delta_{D,t+1} = \left[(1-\delta_{Dt}) d_t / d_{t+1}\right] \delta_{Dt}^{\alpha} + \left(I_t/d_{t+1}\right) \kappa \qquad \alpha, \kappa \in (0,1)$$

$$R_{t+1} = \begin{cases} \left[(1-\delta_{Dt}) d_t / d_{t+1}\right] R_t + (I_t/d_{t+1}) i_t & \text{if FRM,} \\ i_t & \text{if ARM,} \end{cases}$$

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$$I_t = \theta p_t x_{Ht} \qquad \theta \in [0,1]$$

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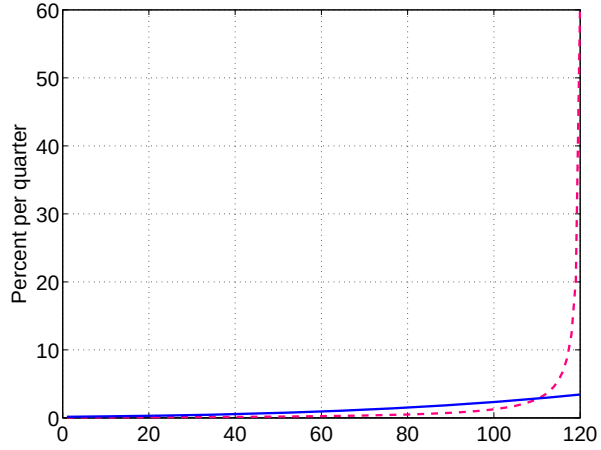
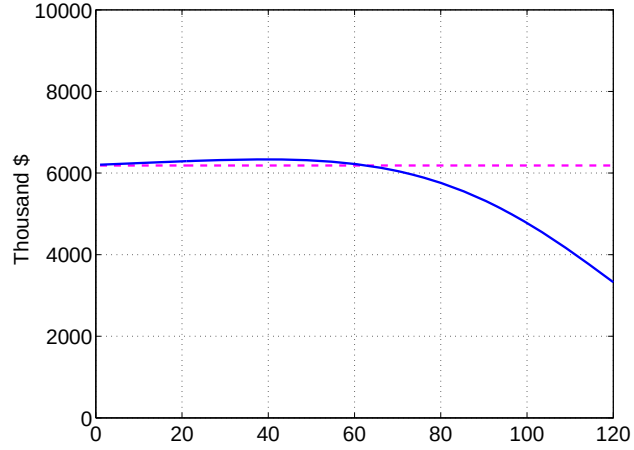
Mortgage (30-yr FRM): model vs calculator

$\alpha = 0.9946,\ \kappa = 0.00162$ ($d_0 = \$250,000,\ i = 9.28\%$)

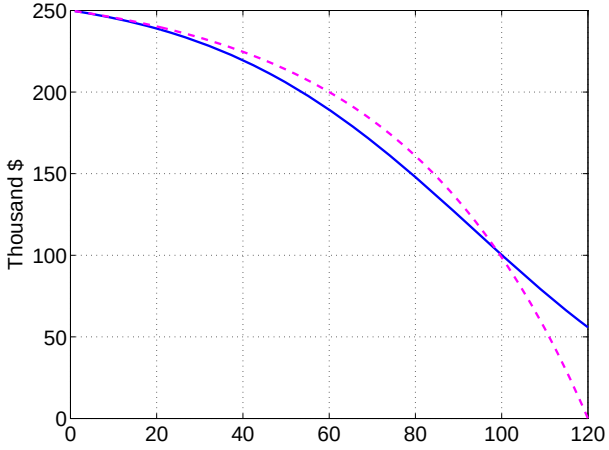
Amortization rate



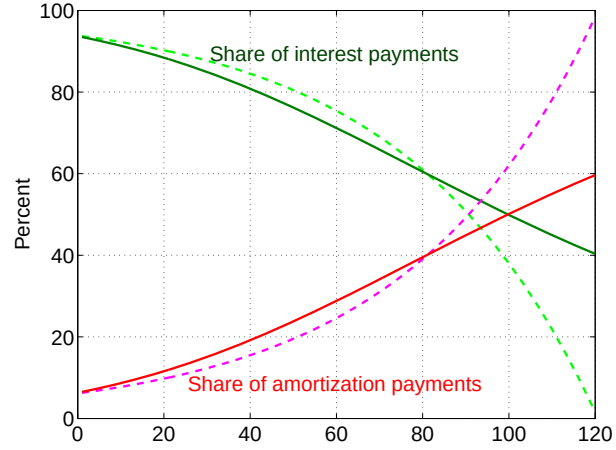
Quarterly payments



Balance



Composition of payments



Model—closing the model

- Government—operates the mortgage market, collects taxes

$$\tau_t = \tau_r r_t k_{Mt} + \tau_w w_t h_{Mt} - \tau_r \delta_M k_{Mt} + m_t/p_t - l_t/p_t$$

- Exogenous VAR(n) process for $Z_t = [z_{Mt}, i_t, \pi_t]$
- State variables: $\mu_t = [Z_t, ..., Z_{t-n}, s_{2t}, ..., s_{Jt}, k_{Mt}, k_{Ht}, d_t, \delta_t, R_t]$

Equilibrium effect of mortgage finance

Shows up only in FOC for x_{Ht}:

$$\underbrace{u_{1t}c_{1t}}_{\text{MU of } c_{Mt}} \quad (1+\tau_{Ht}) \quad = \quad \beta E_t \quad \underbrace{V_{kH,t+1}}_{\substack{\text{Marginal} \\ \text{life-time U} \\ \text{of } k_{H,t+1}}}$$

Time-varying wedge in FOC for x_{Ht}

$$\tau_{Ht} = -\theta \left\{ 1 + \frac{\beta E_t V_{d,t+1}}{u_{1t} c_{1t}} + \frac{\beta \left[\zeta_t (\kappa - \delta_{Dt}^{\alpha}) E_t V_{\delta_D, t+1} + \zeta_t (i_t - R_t) E_t V_{R,t+1} \right]}{u_{1t} c_{1t}} \right\}$$

$$\begin{aligned} V_{dt} = -u_{1t} c_{1t} \frac{R_t + \delta_{Dt}}{1 + \pi_t} &+ \beta \frac{1 - \delta_{Dt}}{1 + \pi_t} E_t V_{d,t+1} \\ &+ \beta \frac{1 - \delta_{Dt}}{1 + \pi_t} \vartheta_t (\delta_{Dt}^{\alpha} - \kappa) E_t V_{\delta_D, t+1} \\ &+ \beta \frac{1 - \delta_{Dt}}{1 + \pi_t} \vartheta_t (R_t - i_t) E_t V_{R,t+1} \end{aligned}$$

$\vartheta_t \approx$ share of new debt ($\theta p_t x_{Ht}$) in total debt
$\zeta_t \approx$ share of old debt in total debt

The V's are derivatives of the value fn w.r.t. to state variables

Time-varying wedge in FOC for x_{Ht}

$$\tau_{Ht} = -\theta \left\{ 1 + \frac{\beta E_t V_{d,t+1}}{u_{1t} c_{1t}} + \frac{\beta \left[\zeta_t (\kappa - \delta_{Dt}^{\alpha}) E_t V_{\delta_D, t+1} + \zeta_t (i_t - R_t) E_t V_{R,t+1} \right]}{u_{1t} c_{1t}} \right\}$$

$$\begin{aligned} V_{dt} = -u_{1t} c_{1t} \frac{R_t + \delta_{Dt}}{1 + \pi_t} &+ \beta \frac{1 - \delta_{Dt}}{1 + \pi_t} E_t V_{d,t+1} \\ &+ \beta \frac{1 - \delta_{Dt}}{1 + \pi_t} \vartheta_t (\delta_{Dt}^{\alpha} - \kappa) E_t V_{\delta_D, t+1} \\ &+ \beta \frac{1 - \delta_{Dt}}{1 + \pi_t} \vartheta_t (R_t - i_t) E_t V_{R,t+1} \end{aligned}$$

Complicated b/c new (i.e., marginal) debt has different amortization rate and interest rate (FRM) than old debt

Time-varying wedge in FOC for x_{Ht}

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Simplifies if either i) new loans are the same as old loans ($\delta_t^{\alpha} = \kappa$ and $R_t = i_t$) or ii) once-and-for-all house purchase ($\zeta_t = \vartheta_{t+1} = 0, \forall t$)

Features of mortgage markets affect τ_{Ht}

$$\tau_{Ht} = -\theta \left\{ 1 + \frac{\beta E_t V_{d,t+1}}{u_{1t} c_{1t}} + \frac{\beta \left[\zeta_t (\kappa - \delta_{Dt}^{\alpha}) E_t V_{\delta_D, t+1} + \zeta_t (i_t - R_t) E_t V_{R,t+1} \right]}{u_{1t} c_{1t}} \right\}$$

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Features of mortgage markets affect τ_{Ht}

FRM vs ARM

$$\tau_{Ht} = -\theta \left\{ 1 + \frac{\beta E_t V_{d,t+1}}{u_{1t} c_{1t}} + \frac{\beta \left[\zeta_t (\kappa - \delta_{Dt}^{\alpha}) E_t V_{\delta_D, t+1} + \zeta_t (i_t - R_t) E_t V_{R,t+1} \right]}{u_{1t} c_{1t}} \right\}$$

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$$R_{t+1} = \begin{cases} \left[(1 - \delta_{Dt}) d_t / d_{t+1} \right] R_t + (l_t / d_{t+1}) i_t & \text{if FRM,} \\ i_t & \text{if ARM,} \end{cases}$$

Features of mortgage markets affect τ_{Ht}

FRM vs ARM

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$$V_{dt} = -u_{1t} c_{1t} \frac{R_t + \delta_{Dt}}{1 + \pi_t} + \beta \frac{1 - \delta_{Dt}}{1 + \pi_t} E_t V_{d,t+1}$$
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Focus on the simplified form $\Rightarrow$ FRM vs ARM through two effects:

i) $E_t[(R_{t+1}+\delta_{D,t+1})/(1+\pi_{t+1})]$ *ii)* $cov_t[u_{1,t+1} c_{1,t+1}, (R_{t+1}+\delta_{D,t+1})/(1+\pi_{t+1})]$

Features of mortgage markets affect τ_{Ht}

FRM vs ARM

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1st effect matters when EH doesn't hold (time-varying term premia in data)

Features of mortgage markets affect τ_{Ht}

FRM vs ARM

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i) $E_t[(R_{t+1} + \delta_{D,t+1})/(1 + \pi_{t+1})]$ *ii)* $cov_t[u_{1,t+1} c_{1,t+1}, (R_{t+1} + \delta_{D,t+1})/(1 + \pi_{t+1})]$

2nd effect constant (CRRA U fn & homoscedastic exo. process)

Features of mortgage markets affect τ_{Ht}

Duration of mortgage contract (α, κ)

$$\tau_{Ht} = -\theta \left\{ 1 + \frac{\beta E_t V_{d,t+1}}{u_{1t} c_{1t}} + \frac{\beta \left[\zeta_t (\kappa - \delta_{Dt}^{\alpha}) E_t V_{\delta_D, t+1} + \zeta_t (i_t - R_t) E_t V_{R,t+1} \right]}{u_{1t} c_{1t}} \right\}$$

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$$\delta_{D,t+1} = \left[(1 - \delta_{Dt}) d_t / d_{t+1} \right] \delta_{Dt}^{\alpha} + (l_t / d_{t+1}) \, \kappa$$

Features of mortgage markets affect τ_{Ht}

Loan-to-value ratio θ

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$\vartheta_t \approx$ share of new debt $(\theta p_t x_{Ht})$ in total debt
$\zeta_t \approx$ share of old debt in total debt

Features of mortgage markets affect τ_{Ht}

Dynamics of i_t and π_t, summarized by VAR(n)

$$\tau_{Ht} = -\theta \left\{ 1 + \frac{\beta E_t V_{d,t+1}}{u_{1t} c_{1t}} + \frac{\beta \left[\zeta_t (\kappa - \delta_{Dt}^{\alpha}) E_t V_{\delta_D, t+1} + \zeta_t (i_t - R_t) E_t V_{R,t+1} \right]}{u_{1t} c_{1t}} \right\}$$

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International facts—dynamics of mortgage rates

- Mortgage rates lead the cycle negatively ... but strength of the lead differs across countries

	Relative std. dev.	Correlations of real GDP in t with a variable in $t+j$: j = -4	-3	-2	-1	0	1	2	3	4
AUS ARM	0.59	-0.29	-0.22	-0.16	-0.03	**0.12**	0.25	0.39	0.48	0.50
BEL FRM	0.89	-0.17	0.01	0.19	0.38	**0.56**	0.63	0.60	0.53	0.41
CAN FRM	0.77	-0.52	-0.41	-0.24	-0.04	**0.19**	0.38	0.45	0.45	0.43
FRA FRM	0.87	-0.10	-0.02	0.10	0.20	**0.30**	0.36	0.35	0.31	0.27
UK ARM	1.29	-0.68	-0.52	-0.31	-0.06	**0.17**	0.36	0.49	0.55	0.56
US FRM	0.55	-0.59	-0.55	-0.46	-0.29	**-0.07**	0.09	0.16	0.21	0.23

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Calibration—elements held fixed across countries

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- Functional forms

$$u(.,.) = \omega \log c + (1-\omega)\log(1 - h_M - h_H); \quad c = c_M^{\psi} c_H^{1-\psi}$$

$$G(.,.) = k_H^{\eta} h_H^{1-\eta}; \quad F(.,.) = k_M^{\lambda} h_M^{1-\lambda}; \quad q(.) = \exp(\sigma(x_{tH} - x_H))$$

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- Parameters (based on U.S. data)
  - $J = 4$, $\phi_j = 0.25\ \forall j$ — GKR, *JPE* 2001
  - $\lambda = 0.283$, $\tau_w = 0.243$ — GRR, *RED* 2011
  - $\delta_M = 0.0248$, $\delta_H = 0.0115$ — to match x_M/k_M, x_H/k_H
  - $\beta = 0.988$, $\omega = 0.472$, $\psi = 0.692$, $\eta = 0.305$, $\tau_r = 0.612$
    to *jointly* match h_M, h_H, k_M/y, k_H/y, $(1-\tau_r)(r-\delta_M)$
  - $\sigma = 6.4$ — to match vol. of res. investment

Calibration—country-specific parameters: United States

- 1984.Q1-2006.Q4
- Estimated VAR(3) process for $Z_t = [z_{Mt}, i_t, \pi_t]$
- Mortgage contract—to match payments on 30-yr FRM 9.28%

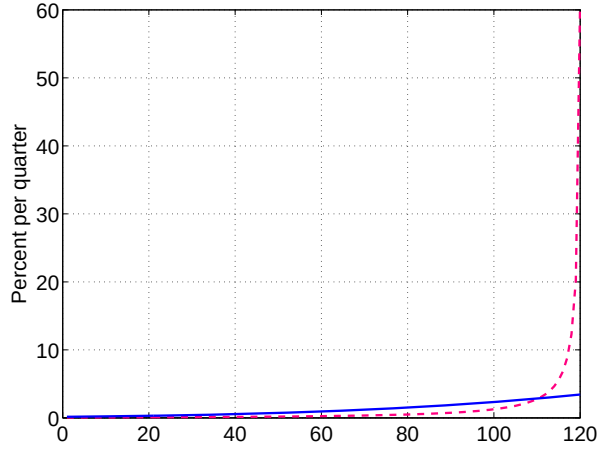
  $\Rightarrow \alpha = 0.9946,\ \kappa = 0.00162$
- LV ratio—to match avrg new mortgage debt to GDP ratio

  $\Rightarrow \theta = 0.78$
  - Compares well with avrg LV ratio from FHFA $= 0.76$
  - Implies steady-state debt-to-GDP ratio $= 1.68$ (data $= 1.95$)
- **Implication**: $\tau_H \approx 0$ in steady state (not targeted!!!)

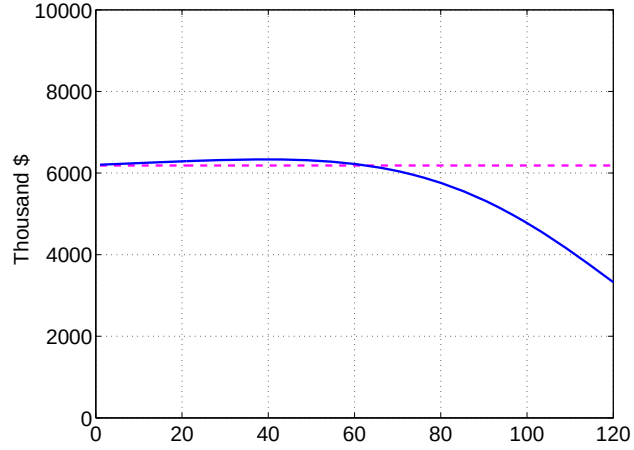
Mortgage (30-yr FRM): model vs calculator

$\alpha = 0.9946,\ \kappa = 0.00162$ ($d_0 = \$250,000,\ i = 9.28\%$)

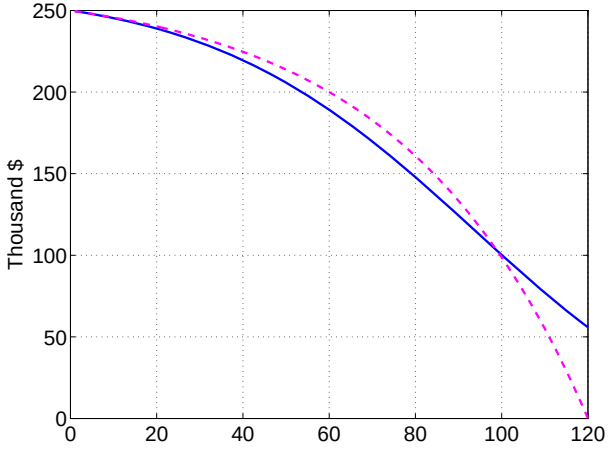
Amortization rate



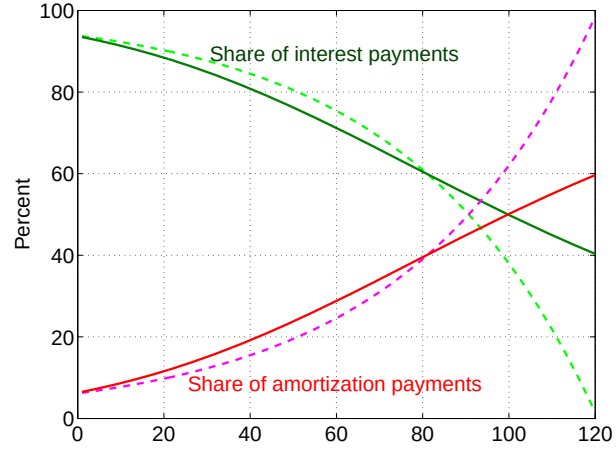
Quarterly payments



Balance



Composition of payments



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Findings—United States

Cyclical behavior of the model economy

v_{t+j}	Rel. st.dev.[b]	Correlations of y in period t with variable v in period $t+j$: j = -4	-3	-2	-1	0	1	2	3	4
i	0.16	-0.22	-0.33	-0.42	-0.41	**-0.29**	-0.13	0.01	0.20	0.34
π	0.30	-0.24	-0.28	-0.34	-0.36	**-0.20**	0.14	0.25	0.22	0.25

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y	1.01	-0.03	0.19	0.48	0.75	**1.00**	0.75	0.48	0.19	-0.03
h_M	0.56	0.10	0.31	0.57	0.76	**0.89**	0.68	0.41	0.07	-0.21
c_M	0.48	-0.21	-0.09	0.13	0.38	**0.70**	0.52	0.38	0.29	0.28
x	4.42	0.07	0.29	0.56	0.78	**0.93**	0.71	0.43	0.10	-0.18

Findings—United States

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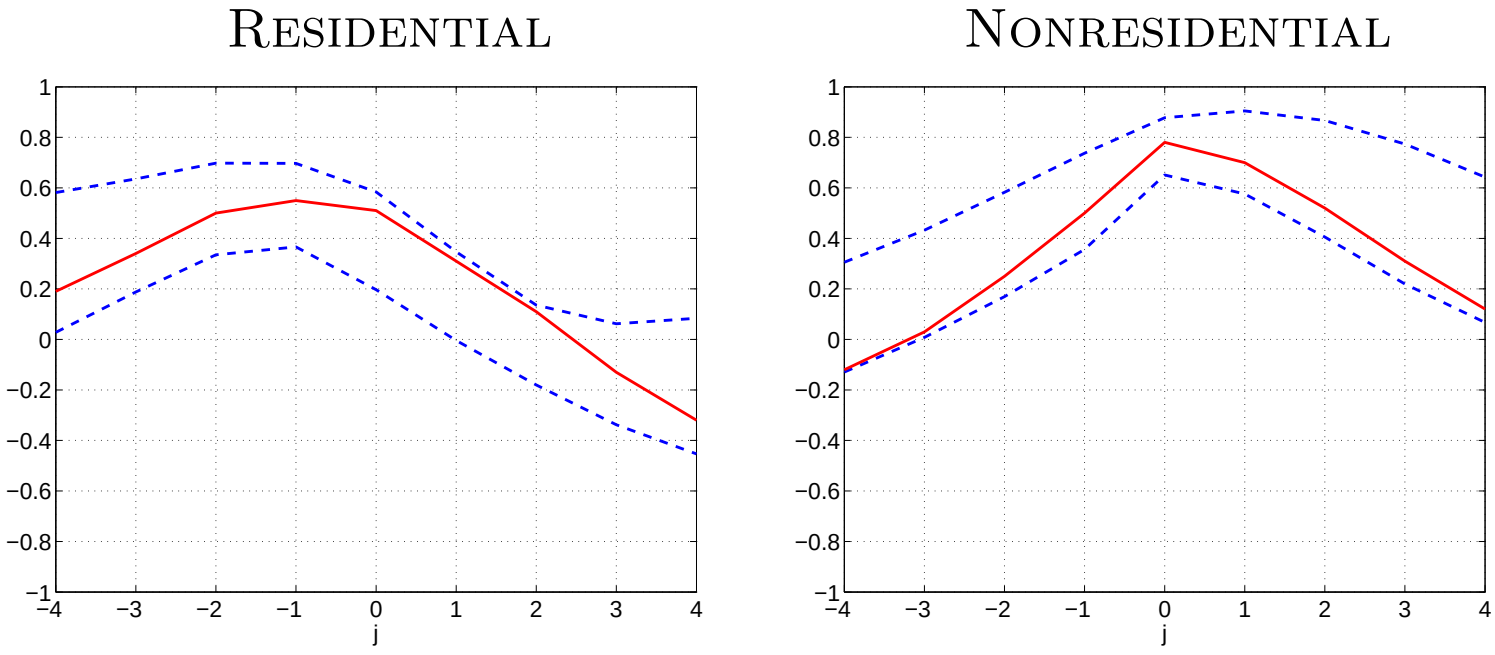
x_H	8.45	0.19	0.34	0.50	0.55	**0.51**	0.31	0.11	-0.13	-0.32
x_M	4.33	-0.12	0.03	0.25	0.50	**0.78**	0.70	0.52	0.31	0.12
τ_H	3.26	-0.21	-0.33	-0.43	-0.43	**-0.32**	-0.17	-0.02	0.18	0.34

Findings—United States



Dashed lines are 95% confidence bands for U.S. data, 84.Q1-06.Q4

Findings—inspecting the role of mortgages

Loan-to-value ratio

v_{t+j}	Rel. st.dev.	Correlations of y in period t with variable v in period $t+j$: j = -4	-3	-2	-1	0	1	2	3	4
U.S. CALIBRATION										
x_H	8.45	0.19	0.34	0.50	0.55	**0.51**	0.31	0.11	-0.13	-0.32
x_M	4.33	-0.12	0.03	0.25	0.50	**0.78**	0.70	0.52	0.31	0.12
τ_H	3.26	-0.21	-0.33	-0.43	-0.43	**-0.32**	-0.17	-0.02	0.18	0.34
NO MORTGAGE FINANCE ($\theta = 0$)										
x_H	0.78	-0.07	0.06	0.30	0.55	**0.84**	0.55	0.37	0.28	0.34
x_M	5.79	0.09	0.28	0.52	0.76	**0.97**	0.74	0.46	0.14	-0.14
NO MORTGAGE FINANCE, LINEAR PPF ($\theta = 0$, $\sigma = 0$)										
x_H	14.66	-0.19	-0.08	0.02	0.20	**0.54**	0.51	0.52	0.48	0.50
x_M	6.32	0.36	0.41	0.54	0.59	**0.52**	0.22	-0.07	-0.29	-0.48

Findings—inspecting the role of mortgages

Duration

v_{t+j}	Rel. st.dev.	Correlations of y in period t with variable v in period $t+j$: j = -4	-3	-2	-1	0	1	2	3	4
U.S. CALIBRATION										
x_H	8.45	0.19	0.34	0.50	0.55	**0.51**	0.31	0.11	-0.13	-0.32
x_M	4.33	-0.12	0.03	0.25	0.50	**0.78**	0.70	0.52	0.31	0.12
τ_H	3.26	-0.21	-0.33	-0.43	-0.43	**-0.32**	-0.17	-0.02	0.18	0.34
15-YEAR FRM ($\alpha = 0.9913$, $\kappa = 0.0083$)										
x_H	5.87	0.18	0.33	0.50	0.55	**0.52**	0.32	0.11	-0.14	-0.33
x_M	4.56	-0.04	0.14	0.38	0.64	**0.91**	0.76	0.54	0.28	0.05
τ_H	2.25	-0.21	-0.32	-0.41	-0.40	**-0.30**	-0.16	-0.01	0.20	0.37

Findings—inspecting the role of mortgages

FRM vs ARM

v_{t+j}	Rel. st.dev.	Correlations of y in period t with variable v in period $t+j$: j = -4	-3	-2	-1	0	1	2	3	4
U.S. CALIBRATION										
x_H	8.45	0.19	0.34	0.50	0.55	**0.51**	0.31	0.11	-0.13	-0.32
x_M	4.33	-0.12	0.03	0.25	0.50	**0.78**	0.70	0.52	0.31	0.12
τ_H	3.26	-0.21	-0.33	-0.43	-0.43	**-0.32**	-0.17	-0.02	0.18	0.34
30-YEAR **FRM**, 4-VARIABLE VAR										
x_H	8.46	0.35	0.49	0.56	0.55	**0.48**	0.31	0.16	-0.04	-0.23
x_M	5.55	-0.08	-0.01	0.23	0.50	**0.80**	0.69	0.50	0.31	0.23
i	0.14	-0.34	-0.48	-0.49	-0.44	**-0.30**	-0.17	-0.05	0.11	0.27
τ_H	2.87	-0.34	-0.49	-0.51	-0.47	**-0.34**	-0.22	-0.10	0.08	0.26
30-YEAR **ARM**, 4-VARIABLE VAR										
x_H	2.59	0.31	0.22	0.04	-0.17	**-0.43**	-0.54	-0.60	-0.60	-0.54
x_M	7.56	0.09	0.20	0.44	0.69	**0.96**	0.77	0.53	0.29	0.13
i	0.25	-0.55	-0.54	-0.48	-0.37	**-0.19**	0.01	0.17	0.32	0.43
τ_H	1.01	-0.26	-0.20	0.02	0.27	**0.61**	0.58	0.59	0.56	0.58

Conclusions (so far)

- Try to understand cyclical dynamics of residential and nonresidential investment
- Available international evidence suggests structure of mortgage markets may play a role
- Construct a tractable business cycle model to quantitatively evaluate this channel
- First results suggest this channel is important
- To do: apply the model to the other countries in our sample

Table 2

Contribution to Weakness in GDP, The Year Before the Recession

Largest in Bold and Boxed

	1949	**1953**	1957	1960	1970	1974	1980	1981	1990	**2001**	Avg	Avg-7
Residential Investment	**30%**	6%	**22%**	**30%**	**20%**	**29%**	**32%**	22%	21%	12%	22%	25%
Durables	19%	18%	20%	12%	20%	24%	26%	10%	**26%**	**23%**	20%	20%
Services	3%	0%	16%	2%	2%	9%	17%	**28%**	2%	8%	9%	11%
Nondurables	7%	7%	0%	8%	11%	21%	10%	8%	8%	7%	10%	9%
Exports	27%	31%	17%	0%	5%	0%	0%	14%	6%	20%	17%	6%
Equipment and Software	15%	4%	0%	0%	7%	0%	15%	0%	25%	19%	14%	7%
Fed Defense	0%	0%	0%	22%	16%	12%	0%	0%	8%	2%	10%	8%
Fed Nondefense	0%	0%	16%	20%	6%	4%	0%	3%	0%	3%	7%	7%
State and Local	0%	**34%**	1%	7%	10%	0%	0%	16%	3%	2%	8%	5%
Structures	0%	0%	8%	0%	2%	2%	0%	0%	1%	3%	3%	2%
TOTAL	-2.8	-0.9	-2.4	-2.3	-2.5	-3.1	-2.7	-2.7	-2.7	-2.4	-2.5	-2.64
Inventories/TOTAL	23%	48%	4%	63%	24%	0%	28%	0%	8%	45%	24%	18%
Imports/TOTAL	14%	50%	5%	2%	20%	0%	0%	14%	0%	0%	10%	6%

back