Cyclical Residential Investment and Mortgages: An International Perspective*

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Abstract

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1 Introduction

2 Model economy

The purpose of the model economy is to parsimoniously summarize and quantitatively assess the effects of some of the features of national mortgage markets on decisions to invest in residential and nonresidential capital over the business cycle. We take the market/home production model of Gomme, Kydland and Rupert (2001) as a benchmark and augment it to include an approximation of mortgage contracts and a stochastic process for mortgage and inflation rates.

Before getting into details, it is worth pointing out three aspects of the model. First, mortgage and inflation rates are exogenous—they follow a joint VAR(n) process with TFP. The model is thus essentially a partial equilibrium model, even though there is a government budget constraint ensuring that the aggregate resource constraint is satisfied. Modeling mortgage and inflation rates as a joint exogenous stochastic process with TFP is motivated by practical considerations: Given our question, it is crucial to reproduce the lead-lag relationship between output (or TFP) and mortgage and inflation rates. Unfortunately, existing literature does not provide a mechanism that would generate the observed lead-lag patterns endogenously.[1]

Second, we do not model the underlying frictions that give rise to mortgages (and to their different types). In the model mortgages are imposed on households by requiring that a fraction of residential investment is financed by debt. Thus, as with cash-in-advance models of money, we use the model to answer a question without explicitly modeling the frictions leading to the very existence of the key asset. The reason for this modeling choice

[1] The difficulty of existing models to generate realistic dynamics of interest rates over the business cycle has been highlighted by, among others, Canzoneri, Cumby and Diba (2007) and Atkeson and Kehoe (2008). **?** shows that reproducing the observed lead-lag patterns of interest and inflation rates in models that endogenize these variables through Taylor-type monetary policy rules requires a time-varying 'wedge' in an Euler equation for bonds that cyclically co-moves with TFP in a particular way. In contrast, standard models account well for the cyclical behavior of the real return on nonresidential capital (Gomme, Ravikumar and Rupert, 2011).

is related to the previous one: endogenizing mortgages through household heterogeneity and borrowing and lending would also endogenize the mortgage rate (to ensure that the market for loanable funds clears). This would, however, lead to the aforementioned problem of generating realistic dynamics of the mortgage rate.

Third, we consider one mortgage market structure at a time. In particular, agents are faced with either FRM (in the case of Belgium, Canada, France, and the United States) or ARM (in the case of Australia and the United Kingdom).

We think of the following exercise as a useful first step towards a more complete analysis of the interaction among mortgage finance, short- and long-term interest rates, and the business cycle. It is worthwhile to explore how well we can account for the cross-country differences in the dynamics of investment data when mortgages and mortgage rates are exogenous, before attempting to endogenize them in a data-consistent way.

2.1 Preferences and technology

There is an infinitely lived representative household and a perfectly competitive representative firm, which operates an aggregate production function. The household has preferences over consumption of a market good produced by the firm c_{Mt}, a good produced at home c_{Ht}, and leisure. Leisure is given as $1 - h_{Mt} - h_{Ht}$, where h_{Mt} is hours supplied to the firm and h_{Ht} is hours spent in 'home production'. Preferences are summarized by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t, 1 - h_{Mt} - h_{Ht}\right), \quad \beta \in (0,1), \tag{1}$$

where $u(.,.)$ satisfies all standard assumptions and c_t is a composite good, given by a constant-returns-to-scale aggregator $c(c_{Mt}, c_{Ht})$.

As in GKR, home hours are combined with a stock of home capital k_{Ht} to produce the home good

$$c_{Ht} = A_H G(k_{Ht}, h_{Ht}), \tag{2}$$

where the production function $G(.,.)$ satisfies all standard assumptions. In contrast to GKR's notion of the home good, which includes output of any economic activity carried out at home, as opposed to in the market, we think of the home good as including only services derived from the use of owner-occupied dwellings. Home capital is thus equivalent to residential structures, and we will refer to it as 'residential capital'. In this context, we think of home hours as time spent on home maintenance and improvements and leisure enjoyed at home, as opposed to in bars and parks, which is $1 - h_{Mt} - h_{Ht}$.

Output of the market good y_t is determined by the aggregate production function

$$y_t = A_{Mt} F(k_{Mt}, h_{Mt}), \tag{3}$$

where A_{Mt} is market TFP and k_{Mt} is market capital, which we will refer to as 'nonresidential capital'.[2] The firm buys market hours and capital services from the household at a wage rate w_t and a capital rental rate r_t, respectively. The market good can be used for consumption, investment in residential capital, x_{Ht}, and investment in nonresidential capital, x_{Mt}. As in Huffman and Wynne (1999) the economy's production possibility frontier (PPF) is nonlinear: for a given y_t, the transformation rate between c_{Mt} and x_{Mt} on one hand and x_{Ht} on the other is $q_t = q(x_{Ht})$, where $q(.)$ is strictly increasing and convex and $q(x_H) = 1$ (i.e., in steady state $q = 1$). Under these assumptions the opportunity cost, in terms of c_{Mt} or x_{Mt}, of increasing x_{Ht} by one unit increases with x_{Ht}. The reason for introducing this nonlinearity into the model is technical: as in portfolio-choice models, at realistic levels of leverage the household's decision to invest in one or the other type of capital becomes very sensitive to small changes in relative rates of return. Making the frontier nonlinear reduces this volatility. On substantive grounds, Huffman and Wynne

[2]Notice that in contrast to A_{Mt}, which is time varying due to shocks, A_H is constant. GKR show that under enough separability in utility and production functions (described in the next section) shocks to A_H do not affect market variables (i.e., hours supplied to firms, consumption of the market good, and accumulation of the two types of capital). This is convenient for calibration as, in contrast to market TFP, home TFP is unobservable.

(1999) argue that a nonlinear PPF reflects some underlying intratemporal costs of moving resources across industries (in our case in and out of residential construction).

Accumulation of nonresidential capital is characterized by a J-period time-to-build (Kydland and Prescott, 1982), where J is an integer greater than one. Specifically, an investment project started in period t becomes a part of productive capital only in period $t+J$. However, the project requires resources throughout the construction period from t to $t+J-1$. In particular, it requires $\phi_j \in [0,1]$ units of investment in period $t+J-j$, where $j \in \{1, ..., J\}$ denotes the number of periods from completion and $\sum_{j=1}^{J} \phi_j = 1$. Let s_{jt} be the number of projects that in period t are j periods from completion. Total nonresidential investment in period t is thus

$$x_{Mt} = \sum_{j=1}^{J} \phi_j s_{jt} \tag{4}$$

and the projects and nonresidential capital evolve as

$$s_{j-1,t+1} = s_{jt}, \quad j = 2, \ldots, J, \tag{5}$$

$$k_{M,t+1} = (1-\delta_M) k_{Mt} + s_{1t}, \tag{6}$$

where $\delta_M \in (0,1)$. Residential capital, in contrast, has only one-period time-to-build, which implies a law of motion

$$k_{H,t+1} = (1-\delta_H) k_{Ht} + x_{Ht}, \tag{7}$$

where $\delta_H \in (0,1)$. Although in actual economies residential construction may also be subject to time-to-build, the lead times are generally shorter than for nonresidential construction (see references in Gomme et al., 2001, for empirical evidence). For our purposes, the difference is more important than the actual lengths.

2.2 Mortgages

When a household takes a mortgage, it has to make regular mortgage payments throughout the life of the mortgage. The household's budget constraint is thus

$$c_{Mt} + x_{Mt} + q_t x_{Ht} = (1-\tau_r) r_t k_{Mt} + (1-\tau_w) w_t h_{Mt} + \delta_M \tau_r k_{Mt} + \frac{l_t}{p_t} - \frac{m_t}{p_t} + \tau_t, \qquad (8)$$

where τ_r is a tax on income from nonresidential capital, τ_w is a tax on labor income, l_t is the nominal value of new mortgage loans, m_t are nominal mortgage payments on outstanding debt, p_t is the aggregate price level (the price of goods in dollars), and τ_t is a lump-sum transfer.[3]

Mortgage payments are determined as

$$m_t = (R_t + \delta_{Dt}) d_t, \qquad (9)$$

where d_t is nominal mortgage debt outstanding, R_t is (as explained below) an *effective* net nominal interest rate on the outstanding debt, and $\delta_{Dt} \in (0,1)$ is an *effective* amortization rate. The variables d_t, R_t, and δ_{Dt} are state variables that evolve recursively as

$$d_{t+1} = (1-\delta_{Dt}) d_t + l_t, \qquad (10)$$

$$\delta_{D,t+1} = \left[(1-\delta_{Dt}) d_t / d_{t+1}\right] \delta_{Dt}^{\alpha} + (l_t / d_{t+1})\, \kappa, \qquad (11)$$

$$R_{t+1} = \begin{cases} \left[(1-\delta_{Dt}) d_t / d_{t+1}\right] R_t + (l_t/d_{t+1}) i_t & \text{if FRM,} \\ i_t & \text{if ARM.} \end{cases} \qquad (12)$$

Here, i_t is the net nominal interest rate (either fixed or adjustable) on new mortgage loans and $\alpha, \kappa \in (0,1)$ are parameters controlling the evolution of the amortization rate so that

[3] Notice that τ_r and τ_w are constant. The reason for introducing these taxes into the model is for calibration purposes, as described below. The lump-sum transfer is time-varying and its role is to ensure that the economy's resource constraint holds.

mortgage payments m_t approximate mortgage payments obtained from actual mortgage calculators. Notice that by combining equations (9) and (10) the evolution of debt can be alternatively written in a more familiar form as $d_{t+1} = d_t + R_t d_t - m_t + l_t$. New loans are determined as

$$l_t = \theta p_t q_t x_{Ht}, \tag{13}$$

where $\theta \in [0, 1]$ is a loan-to-value ratio.[4]

It is worth pausing here to explain these laws of motion and the determination of m_t. Let us consider an individual who has no outstanding mortgage debt and takes a fixed-rate mortgage $l_0 > 0$ in period $t = 0$. The individual does not take any new mortgage debt in later periods (i.e., $l_1 = l_2 = ... = 0$). In period $t = 1$, his outstanding debt is $d_1 = l_0$, the amortization rate at which this debt is reduced is $\delta_{D1} = \kappa$, and the effective interest rate is $R_1 = i_0$. Period-1 mortgage payments are thus $m_1 = (R_1 + \delta_{D1})d_1 = (i_0 + \kappa)l_0$. In period $t = 2$ the outstanding debt becomes $d_2 = (1 - \kappa)l_0$, and is reduced at a rate $\delta_{D2} = \kappa^\alpha > \kappa$. The interest rate R_2 is again equal to i_0 and so on.

Figure 5 provides a numerical example of the evolution of these variables. Here, $l_0 =$ \$250,000, $i_0 = 9.28\%$, $\alpha = 0.9946$, $\kappa = 0.00162$, and one period is equal to one quarter. For comparison, the figure also plots the same variables obtained from a Yahoo mortgage calculator for a U.S. 30-year conventional fixed-rate mortgage in the same amount and with the same interest rate as in our example. The figure shows that the model captures reasonably well, not only qualitatively but also quantitatively, three key features of the conventional mortgage. First, the amortization rate is increasing during the lifetime of the mortgage. Second, mortgage payments based on the calculator are constant; in the model they are approximately constant for the first 70 or so periods (17.5 years). And third,

[4] Only residential investment is financed by debt in the model. This is an approximation supported by the fact that residential investment is substantially more debt-finance dependent than nonresidential investment. For instance, according to U.S. Flow of Funds for 2000, outstanding residential mortgage debt was almost five times as large as outstanding nonresidential mortgage debt, and almost two and a half times as large as the outstanding stock of corporate bonds (the stocks of residential and nonresidential assets were roughly the same).

interest payments are front-loaded: they make up most of the quarterly payments at the beginning of the life of the mortgage and very little towards the end. The values of the parameters α and κ were chosen so as to match the time path of the quarterly payments as well as possible.[5] The parameters α and κ can be chosen to also approximate mortgage payments for mortgages with shorter durations than 30 years (e.g., 15 years as in the case of France). This flexibility makes the model easy to apply across countries.

So far we have only considered the case of an individual taking a mortgage once and for all. Of course, in response to shocks, the representative household adjusts x_t, and thus l_t, every period. In this case, δ_{Dt} and R_t are the effective amortization and interest rates on the economy-wide stock of mortgage debt: $\delta_{D,t+1}$ is the arithmetic average of δ_{Dt}, the effective amortization rate of existing debt, and κ, the initial amortization rate of new debt; and R_{t+1} is the arithmetic average of R_t, the effective interest rate on existing debt, and i_t, the market interest rate on new debt. Indeed, in the case of ARM, $R_t = i_t$ for all t. The adoption of the effective amortization and interest rates considerably reduces the state space, which, in the case of a 30-year mortgage, would (in a quarterly model) contain 120 vintages of mortgage debt. Although with linear approximation methods such a large state space can be handled, the use of the effective rates yields a more transparent characterization of equilibrium conditions. Furthermore, it adds only two (easily calibrated) parameters to the benchmark model.

2.3 Exogenous process and closing the model

As mentioned above, the inflation rate, defined as $\pi_t \equiv \log p_t - \log p_{t-1}$, and the mortgage rate i_t follow a joint VAR(n) process with market TFP

$$z_{t+1} b(L) = \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N(0, \Sigma), \tag{14}$$

[5] When $\alpha = 1$, the share of interest payments in m_t is constant and m_t is declining monotonically throughout the lifetime of the mortgage.

where $z_t = [\log A_{Mt}, i_t, \pi_t]^\top$, $b(L) = I - b_1 L - ... - b_n L^n$ (L being the lag operator), and $\Sigma = BB'$. The choice of L is guided by the data and may differ across countries.

The model is closed by a government budget constraint. The government collects taxes and gives transfers to the household. In addition, it operates an agency that provides mortgage loans. The government's consolidated budget constraint is

$$\tau_t = \tau_r r_t k_{Mt} + \tau_w w_t h_{Mt} - \tau_r \delta_M k_{Mt} + m_t/p_t - l_t/p_t, \tag{15}$$

where the last two terms are the cash inflow of the mortgage-lending agency.

3 Equilibrium Effects of Mortgages

This section characterizes the equilibrium and shows how the equilibrium effects of various features of mortgage markets can be parsimoniously summarized by a wedge in an Euler equation for residential capital.

3.1 Equilibrium

In a dynamic competitive equilibrium the following conditions hold: (i) the representative household solves its utility maximization problem, described below, taking all prices and the lump-sum transfers as given; (ii) r_t and w_t are equal to their marginal products and $q_t = q(x_{Ht})$; and (iii) the government budget constraint (15) is satisfied. The aggregate resource constraint, $c_{Mt} + x_{Mt} + q_t x_{Ht} = y_t$, then holds by Walras' Law.[6]

[6]The equilibrium is computed by incorporating the linear-quadratic method for non-linear constraints (?) into the linear-quadratic method for distorted economies (Kydland, 1989; Hansen and Prescott, 1995). Specifically, after transforming the model so that it is specified in terms of π_t and d_t/p_{t-1} (rather than p_t and d_t), the home production function (2) and the budget constraint (8) are substituted in the period utility function $u(.,.)$, which is then used to form a Lagrangean that has the nonlinear laws of motion (10)-(12) as constraints. This Lagrangean then forms the return function to be quadratically approximated.

It is convenient to write the household's problem as a Bellman equation

$$V\left(s_{1t}, ..., s_{J-1,t}, k_{Mt}, k_{Ht}, d_t, \delta_{Dt}, R_t\right) =$$

$$\max\left\{u\left(c_t, 1-h_{Mt}-h_{Ht}\right) + \beta E_t V\left(s_{1,t+1}, ..., s_{J-1,t+1}, k_{M,t+1}, k_{H,t+1}, d_{t+1}, \delta_{D,t+1}, R_{t+1}\right)\right\}.$$

Here, after substituting the home production function (2), the laws of motion for capital (4)-(7), the budget constraint (8), and the laws of motion for mortgage payments (9)-(13) in the right-hand side of the Bellman equation, the maximization is with respect to h_{Mt}, h_{Ht}, x_{Ht}, and s_{Jt}. There is enough separability in this problem that the variables affecting real mortgage payments (l_t, d_t, δ_{Dt}, R_t, i_t, and π_t) show up only in the first-order condition for x_{Ht}[7]

$$u_{1t}c_{1t}(1-\theta) - \theta\beta E_t\left[\widetilde{V}_{d,t+1} + \zeta_{Dt}(\kappa - \delta_{Dt}^{\alpha})V_{\delta_D,t+1} + \zeta_{Dt}(i_t - R_t)V_{R,t+1}\right] = \beta E_t V_{kH,t+1}, \tag{16}$$

where

$$\zeta_{Dt} \equiv \frac{\frac{1-\delta_{Dt}}{1+\pi_t}\widetilde{d}_t}{\left(\frac{1-\delta_{Dt}}{1+\pi_t}\widetilde{d}_t + \theta x_{Ht}\right)^2}, \quad \widetilde{V}_{d,t+1} \equiv V_{d,t+1}p_t, \quad \text{and} \quad \widetilde{d}_t \equiv d_t/p_{t-1}.$$

The last two expressions on the second line are normalizations that impose stationarity in an environment with nonzero steady-state inflation. For $\theta = 0$, the equilibrium is exactly the same as in GKR as the terms containing $\widetilde{d}_t$, δ_{Dt}, R_t, i_t, and π_t drop out.

If debt was amortized at a constant rate and mortgages had an adjustable interest rate, the last two terms in the square brackets in equation (16) would be zero. In such a case the representative household simply equalizes the marginal benefit of additional housing stock with the marginal cost of financing residential investment—foregone consumption of the market good in period t, weighted by θ, and the present value of debt (i.e., foregone

[7]We adopt the convention of using subscripts to denote partial derivatives. Thus, for example, u_{2t} is the first derivative of the u function with respect to its second argument and V_{Rt} is the first derivative of the V function with respect to R_t.

consumption of the market good in future periods), weighted by $(1-\theta)$.[8] The terms $\zeta_{Dt}(\kappa-\delta_{Dt}^{\alpha})V_{\delta_D,t+1}$ and $\zeta_{Dt}(i_t-R_t)V_{R,t+1}$ appear in the first-order condition because new (i.e., marginal) debt has different amortization and interest rates (the latter only in the case of FRM) than old debt.

The Benveniste-Scheinkman conditions for $V_{kH,t}$ and $\widetilde{V}_{dt}$ are, respectively,

$$V_{kH,t}=u_{1t}c_{2t}A_HG_{1t}+\beta(1-\delta_H)E_tV_{kH,t+1}$$

and

$$\widetilde{V}_{dt}=-u_{1t}c_{1t}\left(\frac{R_t+\delta_{Dt}}{1+\pi_t}\right)+\beta\frac{1-\delta_{Dt}}{1+\pi_t}E_t\left[\widetilde{V}_{d,t+1}+\zeta_{xt}(\delta_{Dt}^{\alpha}-\kappa)V_{\delta_D,t+1}+\zeta_{xt}(R_t-i_t)V_{R,t+1}\right], \tag{17}$$

where

$$\zeta_{xt}\equiv\frac{\theta x_{Ht}}{\left(\frac{1-\delta_{Dt}}{1+\pi_t}\widetilde{d}_t+\theta x_{Ht}\right)^2}.$$

The Benveniste-Scheinkman conditions for $V_{\delta_D,t}$ and V_{Rt} are not crucial for the following discussion and are therefore relegated to the Appendix. Notice again that if debt was amortized at a constant rate and mortgages had an adjustable interest rate, equation (17) would boil down to

$$\widetilde{V}_{dt}=-u_{1t}c_{1t}\left(\frac{R_t+\delta_{Dt}}{1+\pi_t}\right)+\beta\left(\frac{1-\delta_{Dt}}{1+\pi_t}\right)E_t\widetilde{V}_{d,t+1}. \tag{18}$$

Here, mortgage payments per additional unit of real debt are weighted by the marginal utility of the market good and the real value of debt depreciates between periods by $(1-\delta_{Dt})/(1+\pi_t)$. Notice that if the maturity of debt was one period, this condition would simplify further to familiar $\widetilde{V}_{dt}=-u_{1t}c_{1t}(1+R_t)/(1+\pi_t)$, where $R_t=i_{t-1}$.

[8] It is straightforward to show that $V_{dt}<0$.

3.2 Mortgages as a wedge

It is convenient to summarize the equilibrium effects of mortgages as a wedge τ_H in the first-order condition for x_{Ht}. Rearranging equation (16) gives

$$u_{1t}c_{1t}(1+\tau_{Ht}) = \beta E_t V_{kH,t+1},$$

where

$$\tau_{Ht} = -\theta \left\{ 1 + \frac{\beta \left[E_t \widetilde{V}_{d,t+1} + \zeta_t(\kappa - \delta^{\alpha}_{Dt}) E_t V_{\delta_D,t+1} + \zeta_t(i_t - R_t) E_t V_{R,t+1} \right]}{u_{1t}c_{1t}} \right\}.$$

Notice that the wedge acts like an ad-valorem tax on residential investment (it can, however, be both positive and negative, depending on parameter values and exogenous processes). It depends on (i) the loan-to-value ratio θ, (ii) the amortization period (governed by α and κ), (iii) whether mortgages have a fixed or adjustable interest rate, and (iv) the joint dynamics of A_{Mt}, i_t and π_t (through $E_t\widetilde{V}_{d,t+1}$, $E_t V_{\delta_D,t+1}$, and $E_t V_{R,t+1}$).

Consider again the simple example of an individual who has no outstanding mortgage debt and makes a once-and-for-all investment decision in period $t = 0$. In this case, $\zeta_{D0} = 0$, as $\widetilde{d}_0 = 0$, and $\zeta_{xt} = 0$ for $t = 1, 2, ...$, as $x_{Ht} = 0$ in $t = 1, 2, ...$ The wedge thus simplifies to

$$\tau_{Ht} = -\theta \left[1 + \beta E_t \widetilde{V}_{d,t+1}/(u_{1t}c_{1t}) \right]$$

and $\widetilde{V}_{dt}$ is governed by the simplified Benveniste-Scheinkman condition (18). From these two expressions it is easy to see that the wedge depends on the interaction between real mortgage payments and the pricing kernel $\beta(u_{1,t+i}c_{1,t+i})/(u_{1t}c_{1t})$.[9]

[9] Notice that the length of the amortization period affects the wedge even in this simplified expression, as δ_{Dt} is still governed by equation (11), and thus depends on α and κ. Notice also that if new mortgages were priced at par using the model's pricing kernel, $\beta E_t\widetilde{V}_{d,t+1}/(u_{1t}c_{1t})$ would be equal to -1 and the wedge would be equal to zero.

4 Calibration

One period corresponds to one quarter and the functional forms are as follows: the period utility function is $u(.,.) = \omega \log c + (1-\omega)\log(1 - h_M - h_H)$; the consumption aggregator is $c(.,.) = c_M^{\psi} c_H^{1-\psi}$; the home production function is $G(.,.) = k_H^{\eta} h_H^{1-\eta}$; and the market production function is $F(.,.) = k_M^{\lambda} h_M^{1-\lambda}$. These choices ensure that shocks to TFP in the home production function do not affect market variables and can be ignored (Gomme et al., 2001). A_H is thus set identically equal to 1. The function $q(.)$ controlling the curvature of PPF is $\exp(\sigma(x_{tH} - x_H))$, where x_H is steady-state residential investment.

The nature of our experiments is such that the parameters of preferences and technology are kept fixed across countries. Calibration of these parameters is based on the United States, for which most of the required information is available for long enough period (in most cases 1958-2006; see the Appendix for data availability). The values of these parameters are reported in Table 1, panel A. Parameters that vary across countries are those characterizing mortgage markets (θ, α, and κ), steady-state i and π, and the parameters of the VAR process. These parameters are calibrated on a country-by-country basis and the values are summarized in panel B. (the VAR processes are, however, relegated to the Appendix).

As in Gomme et al. (2001), $J = 4$ and $\phi_j = 0.25$ for all j. The share in GDP of nonresidential capital income, λ, and the labor income tax, τ_w, can be measured from the National Income and Product Accounts (NIPA). Based on their average values, λ is set equal to 0.283 and τ_w to 0.243 (Gomme et al., 2011). The depreciation rate δ_M is given by the average ratio of residential investment to residential capital stock and the depreciation rate δ_H by the average ratio of nonresidential structures and equipment & software to the corresponding stock. This yields $\delta_H = 0.0115$ and $\delta_M = 0.0248$, which are higher than the average depreciation rates from the Bureau of Economic Analysis (BEA) tables (see Gomme and Rupert, 2007). This is because our model abstracts from population and TFP growth. Once the average population and per-capita GDP growth rates (0.0037 and 0.0044, respectively) are subtracted from the values of δ_M and δ_H, the depreciation rates are comparable to those from BEA.

As noted in the previous section, in order to have a well-defined steady state, the model needs to be transformed so that it is expressed in terms of an inflation rate rather than the price level. The steady-state inflation rate is set equal to 4.54% per annum, the average inflation rate for the period 1971-2006, a period for which the mortgage interest rate data are available.

In the model, $\theta x_H/y$ is the ratio of new mortgage debt to GDP. In the United States the average of this ratio, based on the flow of home and multifamily unit mortgages, is 0.039.[10] However, as the model abstracts from consumer durable goods, we subtract consumer durables from GDP. For GDP modified this way the above ratio is 0.043, which implies θ equal to 0.78. For comparison, the average loan-to-value ratio based on Federal Housing Finance Agency data is 0.76 (1963-2006, conventional single family newly built home mortgage). As mentioned above, $\alpha = 0.9946$ and $\kappa = 0.00162$. These values are chosen so as to match as well as possible quarterly mortgage payments obtained from a mortgage calculator for $4 \times i = 9.28\%$, the average (1971-2006) annual interest rate for a 30-year conventional fixed-rate mortgage (the same practice of calibrating these two parameters is also used for other countries; more on this below). Given these values, the law of motion (11) implies a steady-state amortization rate of 0.0144. Thus, as in the U.S. economy, mortgage debt in the model economy is amortized at a faster rate than at which the housing stock depreciates. The law of motion for debt (10) then implies a steady-state debt-to-GDP ratio of 1.68. This is about 14% lower than the average of the ratio of the stock of home and multifamily unit mortgages to GDP (adjusted for consumer durables). This discrepancy is largely due to the fact that the model does not fit exactly the time-profile of the amortization rate in the upper-left panel of Figure 5.

[10] In the mortgage data, a part of the Flow of Funds data, 'home' unit is defined as containing 1-4 residential units, whereas 'multifamily' unit is defined as containing 5+ residential units.

Due to shorter data on mortgage flows, for countries other than the U.S. the calibration procedure is reversed—the average debt-to-GDP ratio is used to calculate θ.

The discount factor β, the share of consumption in utility ω, the share of market good in consumption ψ, the share of capital in home production η, and the tax rate on income from nonresidential capital τ_r are calibrated jointly. They are chosen to match the average values of h_M, h_H, k_M/y, k_H/y, and the after-tax real rate of return on nonresidential capital. Given these targets, the values of these parameters are determined by steady-state versions of first-order conditions for h_M, h_H, s_J, x_H and the model equivalent to the after-tax return on nonresidential capital, $(1-\tau_r)(r-\delta_M)$. Gomme and Rupert (2007) report that on average U.S. households spend 25.5% of their available time working in the market and 24% in home production. We assume that half of the home production hours are tied to residential capital in the sense described in the previous section, thus setting $h_H = 0.12$. The average capital-to-GDP ratios are 4.88 for nonresidential capital, measured as the sum of structures and equipment & software, and 4.79 for residential capital (in both cases consumer durables are subtracted from GDP). The average (annual) after-tax net rate of return on nonresidential capital is 5.16% (Gomme et al., 2011). These five targets yield $\beta = 0.988$, $\omega = 0.47$, $\psi = 0.69$, $\eta = 0.30$, and $\tau_r = 0.61$. The tax rate is somewhat higher than what is implied by NIPA (Gomme et al., 2011). This is common in models with disaggregated investment (Gomme et al., 2001).

The parameterization of the exogenous stochastic process is based on point estimates of a VAR model estimated on linearly detrended Solow residual and demeaned interest rate for a conventional 30-year FRM and the inflation rate.[11] The lag length is determined by the multivariate AIC. The point estimates and adjusted R^2s for these regressions

[11] Although the mortgage rate is available from 1971, the process is estimated on the sample 1984.Q1-2006.Q4, which covers a more stable period. The series for the Solow residual is taken from data accompanying Gomme and Rupert (2007). The capital stock used for the construction of the residual is the sum of structures and equipment & software (current cost deflated by the consumption deflator). As capital stock data going far enough are less available for other countries, we follow Cooley and Prescott (1995) and approximate the Solow residual for countries other than the U.S. as $\log A_{Mt} = \log y_t - (1-\lambda)h_{Mt}$, where the U.S. value of $\lambda = 0.283$ is used.

are provided in the Appendix. The curvature parameter σ of the PPF is then chosen by matching the ratio of the standard deviations (for HP-filtered data) of residential investment and GDP, which in the data is 8.4.[12] This yields $\sigma = 6.4$, which implies a standard deviation of q_t (relative to that of ouptut) equal to 2.69. When q_t is measured as the ratio of residential investment and GDP deflators, this standard deviation in the data over the VAR sample period is equal to 1.1. This suggests that additional factors than the rate of transformation (e.g., land availability, downpayment requirements), also play a role in reducing the responsiveness of residential investment to shocks.

We close this section by noting that the above parameterization implies that in steady state (under FRM) the wedge τ_H is equal to -0.465%. Essentially equal to zero. This is an outcome of the model, not a calibration target. Thus, at least in steady-state, the equilibrium allocations and prices are approximately the same as in the GKR model.

5 Findings

[12] This is for single family units, 1984.Q1-2006.Q4.

Appendix A: Data

Data appendix

Australia. REAL VOLUMES: GDP, private GFCF, private GFCF machinery and equipment total, private GFCF nondwelling construction total, private GFCF dwellings total—chained dollars, SA, 1959.Q3-2006.Q4, *Australian Bureau of Statistics, National Accounts*; MORTGAGE RATE: standard variable housing loans lending rate (banks)—1959.Q3-2006.Q4, *Reserve Bank of Australia*; SHORT RATE: 3-month T-bill yield—1960.Q1-2006.Q4, *Global Financial Data*. **Belgium.** REAL VOLUMES: GDP at market prices, GFCF total, GFCF in dwellings, GFCF by enterprises, self-employed workers and non-profit institutions—chained 2006 euros, SA, 1980.Q1-2006.Q4, *BelgoStat Online, National Accounts*; MORTGAGE RATE: fixed rate on loans for house purchasing—1980.Q1-2006.Q4, *Global Financial Data*; SHORT RATE: 3-month T-bill yield—1980.Q1-2006.Q4, *Global Financial Data.* **Canada.** REAL VOLUMES: GDP, residential structures, nonresidential structures, machinery and equipment, single dwellings, multiple dwellings—chained 2002 dollars, SA, *Statistics Canada, National Accounts*, 1961.Q1-2006.Q4, except for single and multiple dwellings, which are for 1981.Q1-2006.Q4; MORTGAGE RATE: conventional mortgage lending rate (5-year term)—1961.Q1-2006.Q4, *Statistics Canada*; SHORT RATE: 3-month T-bill yield—1961.Q1-2006.Q4, *Global Financial Data.* **France.** REAL VOLUMES: GDP, total GFCF, GFCF of non financial enterprises (including uninc. entrep.), GFCF of households (excluding uninc. entrep.)—chained euros, SA, 1971.Q1-2006.Q4, *INSEE, National Accounts*; MORTGAGE RATE: mortgage lending rate—1978.Q1-2006.Q4, *Global Financial Data*; SHORT RATE: money market rate—1971.Q1-2006.Q4, *International Financial Statistics* and *Datastream.* **United Kingdom.** REAL VOLUMES: GDP at market prices, GFCF total, GFCF dwellings, GFCF other new buildings and structures, GFCF transport equipment and other machinery and equipment—chained 2002 pounds, SA, 1965.Q1-2006.Q4, *Office for National Statistics, United Kingdom Economic Accounts*; MORTGAGE RATE: sterling standard variable mortgage rate to households—1965.Q1-2006.Q4, *Bank of England*; SHORT RATE: 3-month T-bill yield—1965.Q1-2006.Q4, *Office for National Statistics.* **United States.** REAL VOLUMES: GDP, private fixed investment, private residential fixed investment—chained 2000 dollars, SA, 1958.Q1-2006.Q4, *FRED*; private fixed investment single family, private fixed investment multifamily, private fixed investment equipment and software, private fixed investment structures, motor vehicles and parts, furniture and household equipment, other durable goods, nondurable goods—chained 2000 dollars, SA, 1958.Q1-2006.Q4, *Bureau of Economic Analysis, National Income and Product Accounts*;[13] ASSETS: private nonresidential, equipment and software, structures, residential, consumer durable goods—current cost, year-end estimates, 1958-2006, *Bureau of Economic Analysis, Fixed Assets Tables, Table 1.1*; MORTGAGES: stocks outstanding—*Federal Reserve Board, Z1 Release, Table L.217*, flows—SA, *Federal Reserve Board, Z1 Release, Table F.217*; loan to value ratio, share of adjustable

[13] Corresponding nominal values used to calculate various ratios are from Bureau of Economic Analysis, National Income and Product Accounts, Tables 1.1.5 and 2.3.5.

rate mortgages[14]—1963-2006 (1982-2006), *Federal Housing Finance Agency, Monthly Interest Rate Survey, Table 10*; Lending rates: 30-year conventional mortgage rate—1971.Q1-2006.Q4, *FRED*; Finance rate on consumer installment loans at commercial banks - new autos (48-month loan), new car average finance rate at auto finance companies, weighted average maturity of new car loans at auto finance companies, finance rate on personal loans at commercial banks—1972.Q1-2006.Q4, *Federal Reserve Board, G19 Release*; Short rate: 3-month T-bill yield—1958.Q1-2006.Q4, *St. Louis FRED.*

Appendix B: Further Benveniste-Scheinkman conditions

For completeness, this Appendix provides the Benveniste-Scheinkman conditions for δ_{Dt} and R_t, respectively,

$$V_{\delta_D,t} = -u_{1t}c_{1t}\left(\frac{\widetilde{d}_t}{1+\pi_t}\right)+\frac{\theta x_{Ht}(\kappa-\delta_{Dt}^{\alpha})+\alpha(1-\delta_{Dt})\delta_{Dt}^{\alpha-1}\left(\frac{1-\delta_{Dt}}{1+\pi_t}\widetilde{d}_t+\theta x_{Ht}\right)}{\left(\frac{1-\delta_{Dt}}{1+\pi_t}\widetilde{d}_t+\theta x_{Ht}\right)^2}\left(\frac{\widetilde{d}_t}{1+\pi_t}\right)\beta E_t V_{\delta_D,t+1}$$

$$-\left(\frac{\widetilde{d}_t}{1+\pi_t}\right)\beta E_t \widetilde{V}_{d,t+1}+\left(\frac{\widetilde{d}_t}{1+\pi_t}\right)\frac{\theta x_{Ht}(i_t-R_t)}{\left(\frac{1-\delta_{Dt}}{1+\pi_t}\widetilde{d}_t+\theta x_{Ht}\right)^2}\beta E_t V_{R,t+1}$$

and

$$V_{Rt} = -u_{1t}c_{1t}\left(\frac{\widetilde{d}_t}{1+\pi_t}\right)+\frac{\frac{1-\delta_{Dt}}{1+\pi_t}\widetilde{d}_t}{\frac{1-\delta_{Dt}}{1+\pi_t}\widetilde{d}_t+\theta x_{Ht}}\beta E_t V_{R,t+1}.$$

Appendix C: VAR estimates

[TABLE 4 HERE]

[14] Both for conventional single family newly built homes mortgages.

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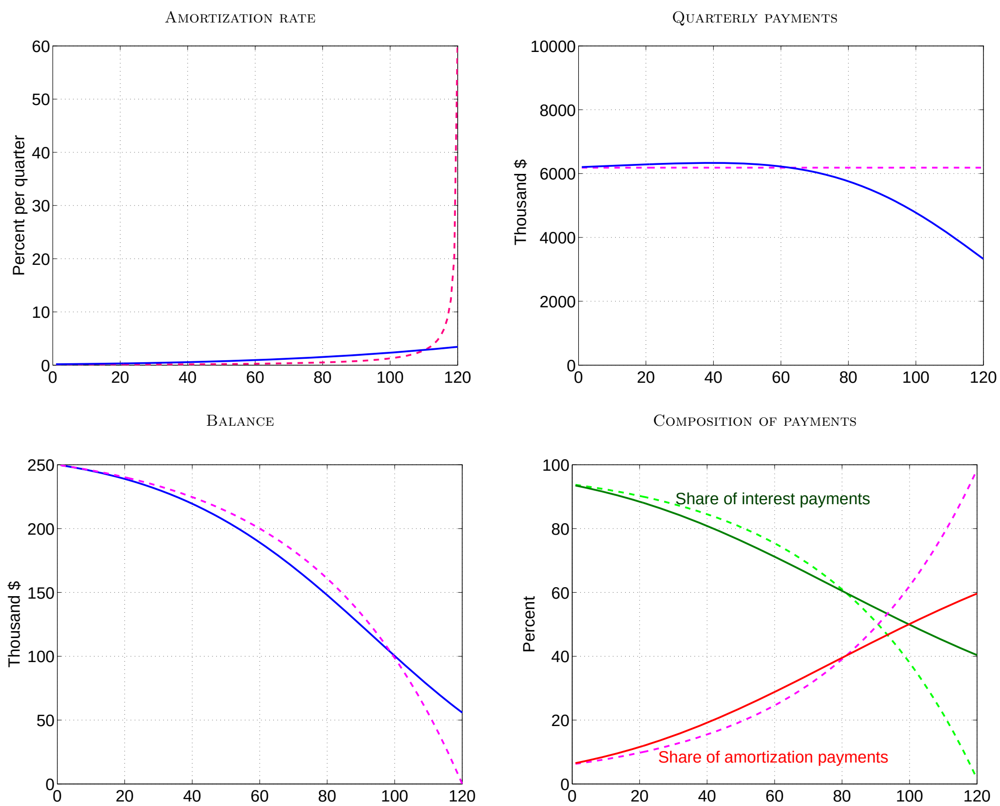

Figure 1: Mortgage: model vs real-world calculator. Solid line=model, dashed line=mortgage calculator. Here, $l_0 = \$250,000$, $4 \times i = 9.28\%$, $\alpha = 0.9946$, and $\kappa = 0.00162$.

Table 1: Calibration

A. Parameters held fixed across countries

Symbol	Value	Definition
Preferences		
β	0.988	Discount factor
ω	0.472	Consumption share in utility
ψ	0.692	Share of market good in consumption
Home technology		
δ_H	0.0115	Depreciation rate
η	0.305	Capital share in production
Time to build		
J	4	Number of project periods
ϕ_j	0.25	Fraction of resources used at stage j
Market technology		
δ_M	0.0248	Depreciation rate
λ	0.283	Capital share in production
σ	6.4	PPF curvature parameter
Tax rates		
τ_w	0.243	Tax rate on labor income
τ_r	0.612	Tax rate on capital income

B. Country-specific parameters

	AUS	BEL	CAN	FRA	UK	US
θ						0.78
α						0.9946
κ						0.00162
i						0.0232
π						0.0113

Table 2: Cyclical behavior of the model economy—U.S. calibration[a]

	Rel.	Correlations of y in period t with variable υ in period $t+j$:								
υ_{t+j}	st.dev.[b]	$j =$ -4	-3	-2	-1	0	1	2	3	4
y	1.01	-0.03	0.19	0.48	0.75	**1.00**	0.75	0.48	0.19	-0.03
h_M	0.56	0.10	0.31	0.57	0.76	**0.89**	0.68	0.41	0.07	-0.21
c_M	0.48	-0.21	-0.09	0.13	0.38	**0.70**	0.52	0.38	0.29	0.28
x_H	8.45	0.19	0.34	0.50	0.55	**0.51**	0.31	0.11	-0.13	-0.32
x_M	4.33	-0.12	0.03	0.25	0.50	**0.78**	0.70	0.52	0.31	0.12
x	4.42	0.07	0.29	0.56	0.78	**0.93**	0.71	0.43	0.10	-0.18
i	0.16	-0.22	-0.33	-0.42	-0.41	**-0.29**	-0.13	0.01	0.20	0.34
π	0.30	-0.24	-0.28	-0.34	-0.36	**-0.20**	0.14	0.25	0.22	0.25
τ_H	3.26	-0.21	-0.33	-0.43	-0.43	**-0.32**	-0.17	-0.02	0.18	0.34

[a] Calibration is as in Tables (1) and (4). The entries are averages for 200 runs of the length of 187 periods each. All quantities are percentage deviations from steady state; the interest and inflation rates, and the wedge, are percentage point deviations from steady state. Before computing the statistics, the artificial series were filtered with HP filter.

[b] Standard deviations are measured relative to that of y; the standard deviation of y is in absolute terms.

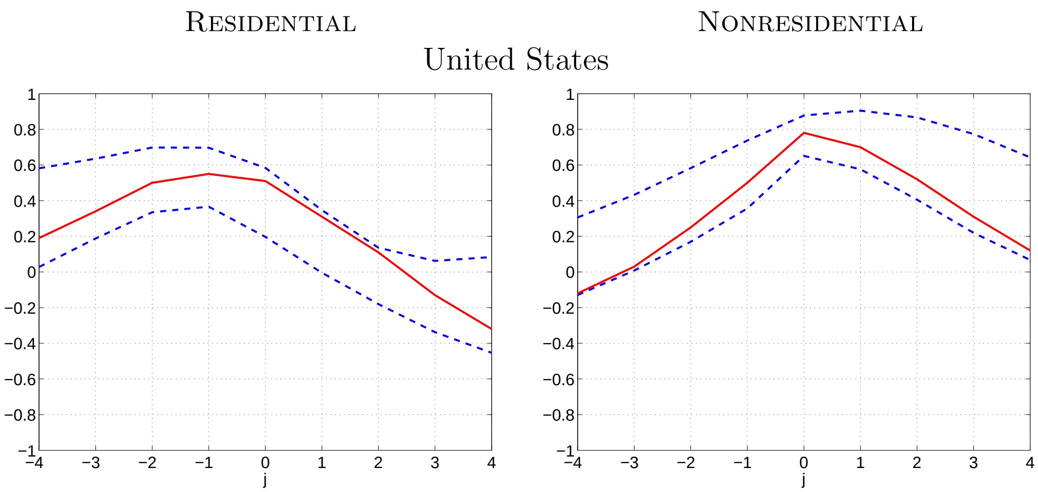

Figure 2: Investment dynamics—model vs data. Solid line=average cross-correlations in simulated data; dashed lines=95% confidence bands for the cross-correlations in actual data obtained by bootstrapping.

Table 3: Inspecting the role of mortgages in investment dynamics

	Rel.	Correlations of y in period t with variable υ in period $t+j$:								
υ_{t+j}	st.dev.[a]	j = -4	-3	-2	-1	0	1	2	3	4
U.S. CALIBRATION[b]										
x_H	8.45	0.19	0.34	0.50	0.55	**0.51**	0.31	0.11	-0.13	-0.32
x_M	4.33	-0.12	0.03	0.25	0.50	**0.78**	0.70	0.52	0.31	0.12
τ_H	3.26	-0.21	-0.33	-0.43	-0.43	**-0.32**	-0.17	-0.02	0.18	0.34
NO MORTGAGE FINANCE ($\theta = 0$)[c]										
x_H	0.78	-0.07	0.06	0.30	0.55	**0.84**	0.55	0.37	0.28	0.34
x_M	5.79	0.09	0.28	0.52	0.76	**0.97**	0.74	0.46	0.14	-0.14
NO MORTGAGE FINANCE, LINEAR PPF ($\theta = 0$, $\sigma = 0$)[c]										
x_H	14.66	-0.19	-0.08	0.02	0.20	**0.54**	0.51	0.52	0.48	0.50
x_M	6.32	0.36	0.41	0.54	0.59	**0.52**	0.22	-0.07	-0.29	-0.48
15-YEAR FRM ($\alpha = 0.9913$, $\kappa = 0.0083$)										
x_H	5.87	0.18	0.33	0.50	0.55	**0.52**	0.32	0.11	-0.14	-0.33
x_M	4.56	-0.04	0.14	0.38	0.64	**0.91**	0.76	0.54	0.28	0.05
τ_H	2.25	-0.21	-0.32	-0.41	-0.40	**-0.30**	-0.16	-0.01	0.20	0.37
30-YEAR ARM[d]										
x_H	2.55	0.35	0.28	0.09	-0.13	**-0.42**	-0.54	-0.61	-0.61	-0.55
x_M	7.07	0.03	0.11	0.36	0.63	**0.93**	0.76	0.52	0.28	0.15
τ_H	1.11	-0.24	-0.17	0.07	0.34	**0.69**	0.65	0.65	0.59	0.58

[a] Standard deviations are measured relative to that of y; the standard deviation of y is in absolute terms.
[b] 30-year FRM, calibration as in Tables (1) and (4).
[c] Here i and π do not distort decisions, but still form a part of the underlying probability space as the household still faces the stochastic process (4). i and π are thus still used to forecast A_M.
[d] 3-month U.S. T-bill rate is used to proxy ARM interest rate. A VAR(4) process for TFP, the T-bill rate, and the inflation rate is estimated and this process replaces in the model the VAR(3) process for the FRM interest rate.

Table 4: Vector autoregressions for exogenous variables—point estimates

United States

	lag 1			lag 2			lag 3		
A_{Mt}	0.933	-0.543	-0.283	0.118	-0.070	0.183	-0.147	0.633	0.117
i_t	0.023	0.953	0.020	-0.016	-0.134	0.036	0.036	-0.011	0.043
π_t	0.021	0.431	0.246	0.111	-0.249	0.164	-0.084	-0.197	0.187

Lower diagonal elements of B: $B_{11} = 0.0049$, $B_{21} = 0.0002$, $B_{22} = 0.0009$, $B_{31} = -0.0011$, $B_{32} = 0.0009$, $B_{33} = 0.0026$.
Sample 1984.Q1-2006.Q4; A_{Mt} is in logs and linearly detrended, annual i and π are divided by 400 and demeaned; number of lags determined by a multivariate AIC; goodness of fit ($\overline{R}^2$): eq. 1 = 0.88, eq. 2 = 0.95, eq. 3 = 0.22.